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Vertex-to-edge detour distance of some standard graphs

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Abstract
In this Research Paper we discuss vertex-to-edge detour distance of some graphs and prove some theorems related to vertex to edge detour distance. Also we discuss detour radial graph of some standard graphs and we found the vertex to edge detour distance of some standard graphs.
Keywords:- vertex and e an edge , Jahangir graph , Splitting graph , Splitting graph , Helm graph , bistar graph , Ladder graph & Conclusion

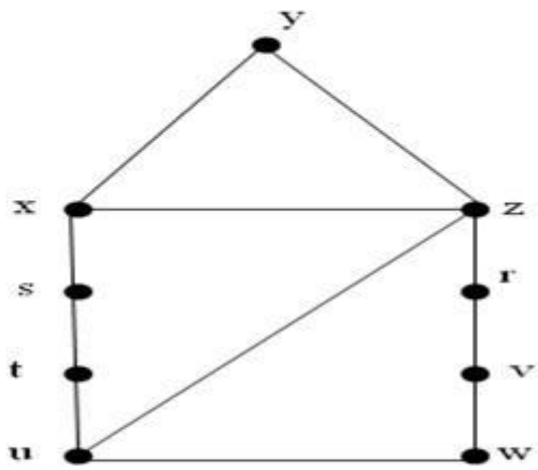
Vertex to Edge detour distance of some standard graphs :-
In this section , we discuss vertex-to-edge detour distance of some graphs and proved some theorems related to vertex to edge detour distance.

Definition
Let u be a vertex and e an edge in a connected graph G . A vertex-to-edge $u - e$ path P is a $u - v$ path, where v is a vertex in e such that P contains no vertices of e other than v . The vertex-to-edge detour distance $D(u,e)$ is the length of a longest $u - e$ path. A $C_1(G)$ of G . Path of length $D(u,e)$ is called a vertex-to-edge $u - e$ detour or simply $u - e$ detour. For our convenience a $u - e$
path of length $d(u,e)$ is called a vertex-to-edge u-e geodesic or simply $u - e$

geodesic.

Example

Consider the following graph



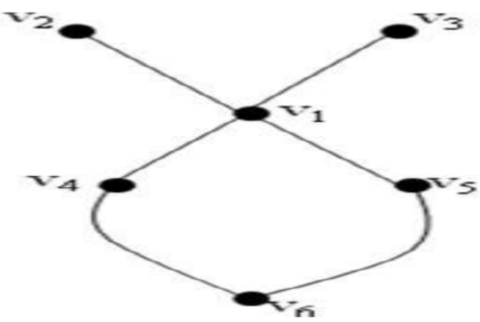
For the vertex u and the edge $e = \{v, w\}$, the paths $P_1: u, w$, $P_2: u, z, r, v$ and $P_3: u, t, s, x, y, z, r, v$ are $u-e$ paths, while the paths $Q_1: u, w, v$ and Q_2: u, z, r, v, w are not $u-e$ paths: Now the vertex-to-edge distance $d(u, e) = 1$and the vertex-to-edge detour distance $D(u, e) = 7$. Thus the vertex-to-edge detour distance is different from the vertex-to-edge distance. Also P_3 is a $u-e$ detour and P_1 is a $u-e$ geodesic. Since the length of a $u-e$ path between a vertex u and an edge e in a graph G of order n is at most $n-2$.

Definition

For a Graph G the splitting graph $S'(G)$ of a graph G is obtained by adding new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.

Example

The following is an example of splitting graph with 6 vertices

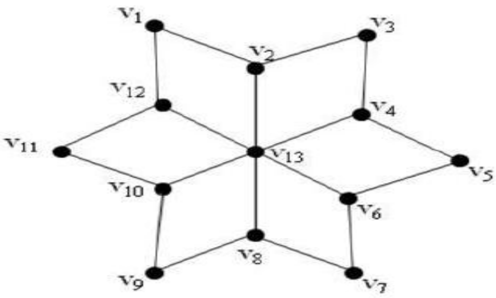


Definition

Jahangir graph $J_{n,m}$ for $m \geq 3$ is a graph on $nm+1$ vertices, (i.e) a graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm} .

Example

The following is an example for Jahangir graph $J_{2,6}$.

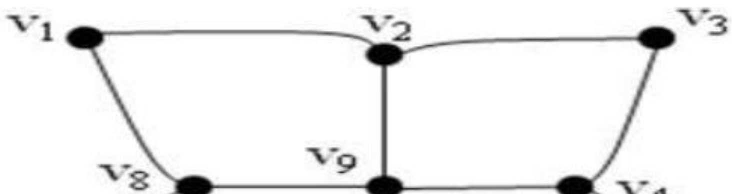


Theorem

Let u be a vertex and e an edge in Jahangir graph such that the vertex u is not in the end vertex of an edge e . Then the vertex to edge detour distance in Jahangir graph $J_{2,m}$ is $2m-1$.

Proof:

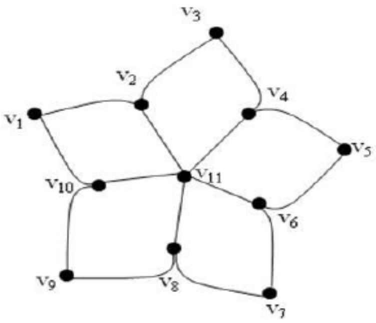
Let us prove the theorem by induction on the second index m of $J_{2,m}$. Let $m=4$, Then $J_{2,4}$ is a Jahangir graph with 9 vertices and is of the form,



The vertex to edge detour eccentricity of the vertices v_1 , v_3 , v_5 ,v_7 is 7 and the vertex to edge detour eccentricity of the vertices v_2 , v_4 , v_6 ,v_8 is 6.The vertex to edge detour radius of $J_{2,4}$ is 6 and the vertex to edge detour diameter of $J_{2,4}$ is 7.

Then the vertex to edge detour distance of $J_{2,4}$ is 7 (i,e) $D(u,e)=2n-1$.

Let $n = 5$, Then $J_{2,5}$ is a Jahangir graph with 11 vertices and is of the form.

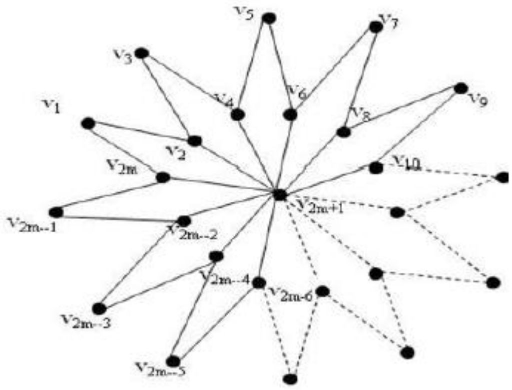


In $J_{2,5}$,the vertex to edge detour eccentricity of the vertices v_1 , v_3 , v_5 , v_7 , v_9 ,v_{11} is 9 and the vertex to edge detour eccentricity of the vertices v_2 , v_4, v_6 , v_8 ,v_{10} is 8. The vertex to edge detour radius of $J_{2,5}$ is 8 and the vertex to edge detour diameter of $J_{2,5}$ is 9.

Hence the vetex to edge detour distance of the Jahangir graph $J_{2,5}$ is 9 (i.e) $D\ (u\ ,e)$ of $J_{2,5}$ is $2n-1$.

Therefore the theorem is true for $m = 4$, $m = 5$. Let us assume that the theorem is true for the second index $m-1$.(i.e) The vertex to edge detour distance in $J_{2,\ m-1}$ is $2m-3$.

Now to prove the theorem is true for the second index m . Let $J_{2,m}$ is a Jahangir graph with $2m+1$ vertices and is of the form,



From the above Jahangir graph the vertex to edge detour eccentricity of the vertices $v_1, v_3, v_5, \ldots\ldots v_{2m-1}, v_{2m+1}$ is $2m-1$. The vertex to edge detour eccentricity of the vertices $v_2, v_4, v_6, \ldots\ldots v_{2m-2}, v_{2m}$ is $2m-2$.

The vertex to edge detour radius of $J_{2,m}$ is $2m-2$. The vertex to edge detour diameter of $J_{2,m}$ is $2m-1$. Therefore the vertex to edge detour distance of $J_{2,m}$ is $2m-1$.

Hence the theorem.

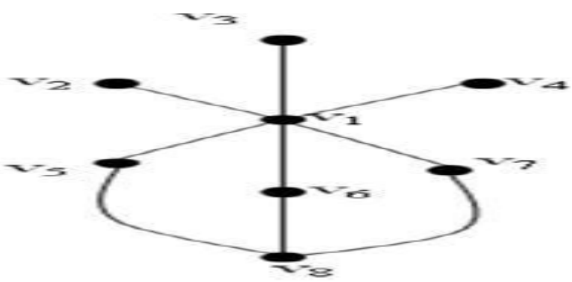
Theorem

Let u be a vertex and e an edge in Splitting graph $S'(K_{1,n})$, such that the vertex u is not in the end vertex of an edge e. Then the vertex to edge detour distance of Splitting graph $S'(K_{1,n})$ is 3.

Proof:

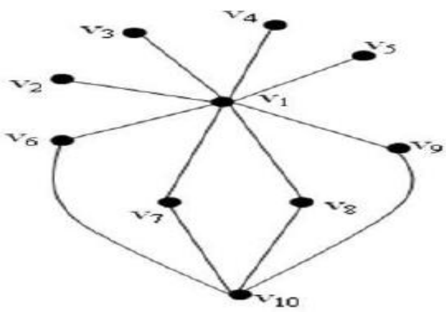
Let us prove the theorem by induction on the second index n of $S'(K_{1,n})$

Let $n = 3$, Then $S'(K_{1,3})$ is a Splitting graph with 8 vertices and is of the form,



The vertex to edge detour eccentricity of the vertices v_1, v_8 is 2 and the vertex to edge detour eccentricity of the vertices $v_2, v_3, v_4, v_5, v_6, v_7$ is 3. The vertex to edge detour radius of $S'(K_{1,3})$ is 2 and the vertex to edge detour diameter of $S'(K_{1,3})$ is 3. Then the vertex to edge detour distance of $S'(K_{1,3})$ is 3 (i.e) $D(u,e) = 3$.

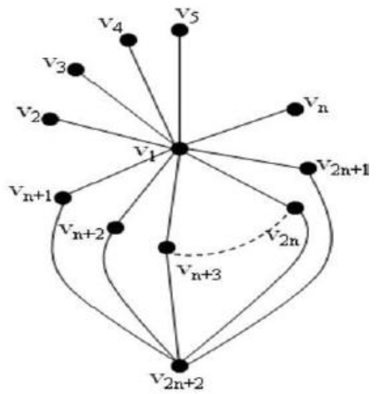
Let $n = 4$. Then $S'(K_{1,4})$ is a Splitting graph with 10 vertices and is of the form.



In $S'(K_{1,4})$,the vertex to edge detour eccentricity of the vertices v_1 , v_{10} is 2 and the vertex to edge detour eccentricity of the vertices v_2 , v_3 , v_4 , v_5 , v_6 , v_7 , v_8 , v_9 is 3. The vertex to edge detour radius of $S'(K_{1,4})$ is 2 and the vertex to edge detour diameter of $S'(K_{1,4})$ is 3.Hence the vertex to edge detour distance of the Splitting graph $S'(K_{1,4})$ is 3 (i.e) $D(u,e)$ of $S'(K_{1,4})$ is 3.

Therefore the theorem is true for $n = 3, n = 4$. Let us assume that the theorem is true for the second index $n-1$.The vertex to edge detour distance of $S'(K_{1,n-1})$ is 3.

Now to prove the theorem is true for the second index n . Let $S'(K_{1,n})$ is a Splitting graph with $2n+2$ vertices and is of the form,



From the above Splitting graph the vertex to edge detour eccentricity of the vertices $v_1 , v_5 ,......v_{2n-1} , v_{2n+2}$ is 2. The vertex to edge detour eccentricity of the vertices $v_2 , v_3 , v_4 ,......v_n , v_{n+1} , v_{n+2} , v_{n+3} ....v_{2n+1}$ is 3. The vertex to edge detour radius of $S'(K_{1,n})$ is 2. The vertex to edge detour diameter of $S'(K_{1,n})$ is 3. Therefore the vertex to edge detour distance of $S'(K_{1,n})$ is 3.

Hence the theorem.

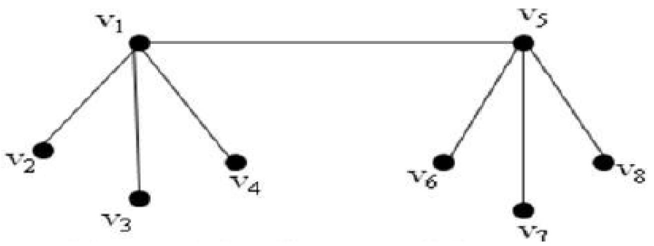
Theorem

Let u be a vertex and e an edge in bistar graph $B_{n,n}$ such that the vertex u is not in the end vertex of an edge e . Then the vertex to edge detour distance in bistar graph $B_{n,n}$ is 2 .

Proof:

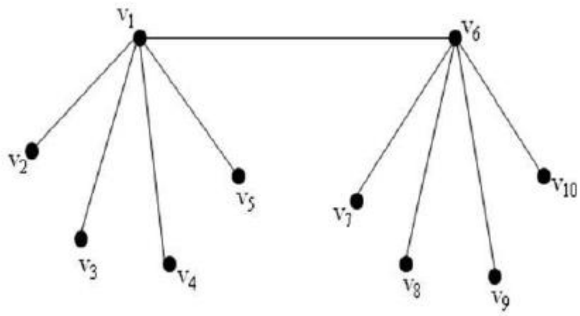
Let us prove the theorem by induction on n . Consider $B_{n,n}$ is a bistar graph having $2n+2$ vertices.

Let $n=3$, Then $B_{3,3}$ is a bistar graph with 8 vertices and is of the form,



The vertex to edge detour eccentricity of the vertices v_1 , v_5 is 1 and the vertex to edge detour eccentricity of the vertices $v_2 , v_3 , v_4 , v_6 , v_7 ,v_8$ is 2.The vertex to edge detour radius of $B_{3,3}$ **3** is 1 and the vertex to edge detour diameter of $B_{3,3}$ is 2. Then the vertex to edge detour distance $B_{3,3}$ is 2 (i,e) $D(u,e)=2$.

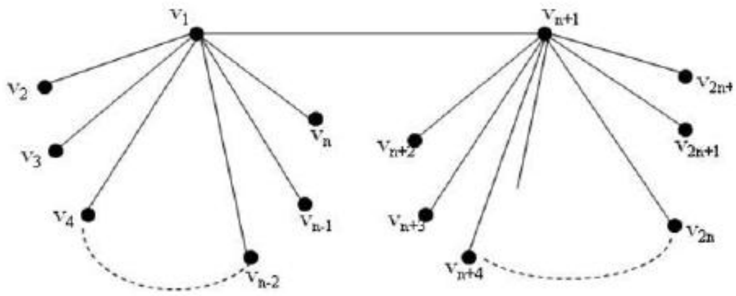
Let $n=4$, Then $B_{4,4}$ is a bistar graph with 10 vertices and is of the form.



In $B_{4,4}$,the vertex to edge detour eccentricity of the vertices v_1 , v_6 is 1 and the vertex to edge detour eccentricity of the vertices v_2 , v_3 , v_4 , v_5 , v_7 , v_8 , v_9 , v_{10} ,v_{11} is 2. The vertex to edge detour radius of $B_{4,4}$ is 1 and the vertex to edge detour diameter of $B_{4,4}$ is 2.Hence the vetex to edge detour distance of the bistar graph $B_{4,4}$ is 2 (i.e) D(u,e) of $B_{4,4}$ is 2.

Therefore the theorem is true for $n = 3, n = 4$. Let us assume that the theorem is true for $n-1$ (i.e) for $B_{n-1,\, n-1}$. The vertex to edge detour distance of $B_{n-1,\, n-1}$ is 2.

Now to prove the theorem is true for n . Let $B_{n,n}$ is a bistar graph with $2n+2$ vertices and is of the form,



From the above Bistar graph $B_{n,n}$ the vertex to edge detour eccentricity of the vertices v_1 , v_{n+1} is 1. The vertex to edge detour eccentricity of the vertices v_2 , v_3 , v_4 , v_5 , v_6 ,.......v_{2n} , v_{2n+1} ,v_{2n+2} is 2.

The vertex to edge detour radius of $B_{n,n}$ is 1. The vertex to edge detour of $B_{n,n}$ is 2. Therefore the vertex to edge detour distance of Bistar graph $B_{n,n}$ is 2.

Hence the proof.

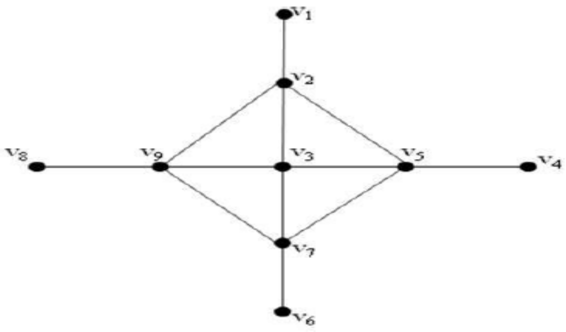
Theorem

Let u be a vertex and e an edge in Helm graph H_n such that the vertex u is not in the end vertex of an edge e . Then the vertex to edge detour distance in Helm graph H_n is n+1.

Proof:

Let us prove the theorem induction on the number of vertices of Helm graph H_n . Consider H_n is a Helm graph having $2n+1$ vertices.

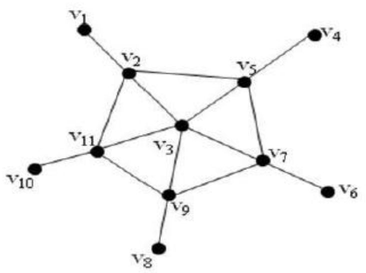
Let $n=4$, Then H_4 is a Helm graph with 9 vertices and is of the form,



The vertex to edge detour eccentricity of the vertices v_1 , v_4 , v_6 ,v_8 is 5 and the vertex to edge detour eccentricity of the vertices v_2 , v_3 , v_5 , v_7 ,v_9 is 4.

The vertex to edge detour radius of H_4 is 4 and the vertex to edge detour diameter of H_4 is 5. Then the vertex to edge detour distance of H_4 is 5 (i,e) $D(u\ ,e)=5$.

Let $n=5$, Then H_5 is a Helm graph with 11 vertices and is of the form.



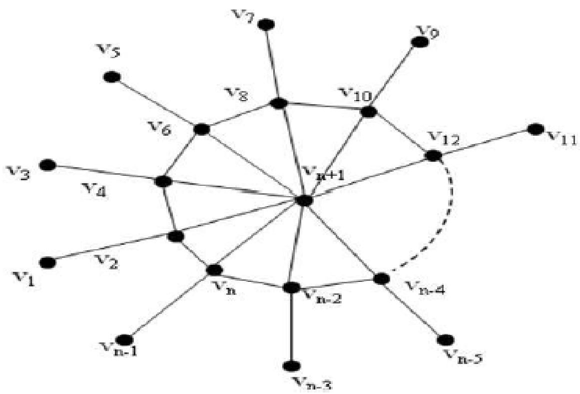
In, H_5 the vertex to edge detour eccentricity of the vertices v_1 , v_4 , v_6 , v_8 ,v_{10} is 6 and the vertex to edge detour eccentricity of the vertices v_2 , v_3 , v_5 , v_7 , v_9 ,v_{11} is 5. The vertex to edge detour radius of H_5 is 5 and the vertex to edge detour diameter of H_5 is 6.

Hence the vertex to edge detour distance of the Helm graph H_5 is 6 (i.e) $D(u, e)$ of H_5 is 6.

Therefore the theorem is true for $n = 4$, $n = 5$. Let us assume that the theorem is true for $n-1$ vertices. The vertex to edge detour distance of H_{n-1} is n.

Now to prove the theorem is true for H_n .

Let H_n is a Helm graph with $2n+1$ vertices and is of the form,



From the above Helm graph H_n the vertex to edge detour eccentricity of the vertices v_1 , v_3 , v_5 ,.......v_{n-1} is $n+1$. The vertex to edge detour eccentricity of the vertices v_2 , v_4 , v_6 ,.......v_n ,v_{n+1} is n .The vertex to edge detour radius of H_n is n . The vertex to edge detour of H_n is $n+1$. Therefore the vertex to edge detour distance of Helm graph H_n is $n+1$.

Hence the proof.

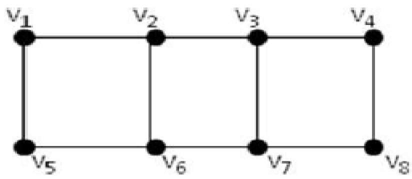
Theorem

Let u be a vertex and e an edge in Ladder graph L_n such that the vertex u is not in the end vertex of an edge e . Then the vertex to edge detour distance in Ladder graph L_n is 2n-2.

Proof:

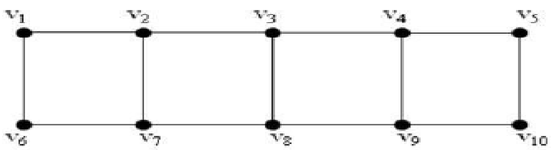
Let us prove the theorem by induction on the index n of the Ladder graph L_n .Consider L_n is a Ladder graph having $2n$ vertices .

Let $n = 4$, Then L_4 is a Ladder graph with 8 vertices and is of the form,

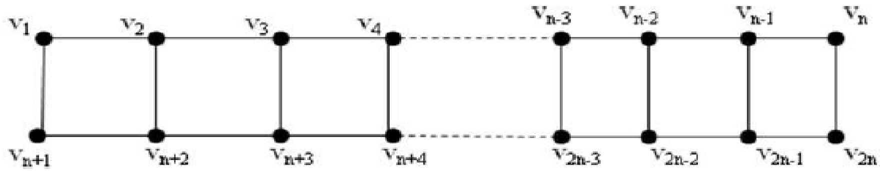


The vertex to edge detour eccentricity of the vertices v_1 , v_2 , v_3 , v_4 , v_5 , v_6 , v_7 ,v_8 is 6.The vertex to edge detour radius and the vertex to edge detour diameter of L_4 is 6. Then the vertex to edge detour distance L_4 is 6 (i,e) $D(u\ ,e) = 6$.

Let $n = 5$, Then L_5 is a Ladder graph with 10 vertices and is of the form.



In L_5 ,the vertex to edge detour eccentricity of all vertices is 8 . The vertex to edge detour radius and the vertex to edge detour diameter of L_5



From the above Ladder graph L_n , the vertex to edge detour eccentricity of all vertices is $2n-2$. The vertex to edge detour radius and The vertex to edge detour diameter of L_n is $2n-2$. Therefore the vertex to edge detour distance of Ladder graph L_n is $2n-2$.

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