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Building A Cylindrical Container Mathematics In Everyday Life Tangible Means, Visual Considerations Or Formal Computations?

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ABSTRACT: *This paper presents activities for teaching the topic of a cylinder and their contribution to the enhancement of comprehension and thinking of abstract geometry. The activities illustrate the transition from 2-D nets of solids, a cylinder in this case. This net of solids is given on a bristol sheet to be used for forming the cylinder, the 3-D solid, while using computations and visual illustrations. The objective is to improve the spatial visualisation and ability which affect the daily life of each and everyone of us. These activities are suitable for those who learn geometry in the higher grades of elementary school, pre-service teachers and in service teachers.*

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I. THEORETICAL BACKGROUND

The National Council of Teachers of Mathematics defined principles and tools for mathematics teaching, underscoring the importance of developing visualization as a tool for solving mathematical problems (NCTM, 2000). This develops, among others, the ability to investigate shapes and solids and contributes to the promotion of spatial perceptions (Patkin & Sarfaty, 2012). Researchers argue that solid geometry is usually learnt with an emphasis on the formal aspect and less on visual view, despite the acknowledgement that visual ability is important for mathematics learning. Teaching that integrates both formal and visual aspects helps learners to cope with geometric assignments although they are included in the curriculum only to small extent (Walker, Winner, Hetland, Simmons & Goldsmith, 2011). Studies of the relation between visual ability and abstract geometric thinking found that these factors facilitate the learning of geometry (Seago, Driscoll & Jacobs, 2010; Walker et al., 2011). Battista and Clements (1998) attribute great importance to the development of strategies through appropriate learning assignments that enhance the building of cognitive framework required for understanding volume measurement and volume determination formulas.

II. INTRODUCTION

The paper relates to the building of a cylinder which is a 3-D solid comprised of two congruent circles positioned in parallel planes and of all the sections connecting these circles. The two circles are referred to as the cylinder bases and the straight cylinder – has a rectangle-shaped lateral surface. In a less formal description, the cylinder has two bases which are congruent and parallel circles and a 'stretched' lateral surface surrounding them. A straight cylinder is a cylinder whereby the section which connects the centre of the basis is perpendicular to the planes of the bases. One can intuitively explain that in a straight cylinder which stands on one of its bases, the lateral surface is in an upright position and the bases are located exactly one above the other. Children in the kindergarten and elementary school engage only in straight cylinders.

When teaching the topic of cylinders, pupils should learn to calculate the area of the surface, the area of the lateral surface and the volume of the cylinder whose dimensions are given by means of numbers and algebraic. Moreover, it is important to discuss the transfiguration of the area of the cylinder surface as a result of addition and multiplication changes in the length of the height and the radius. For example, in cases where the lengths of the height and the radius are multiplied times 2. Learners should obviously know to draw a network of solids, i.e. a cylinder. It is appropriate to engage in problems which integrate computations with a cylinder with facts studied in earlier stages of the 7^{th}-8^{th} grades, including conversion of sizes and implementation of the Pythagorean theorem in space with a cylinder. It is also recommended applying demonstrative examples and illustration aids. In order to assist the learners to succeed in solving the problems dealing with this topic they should engage in questions taken from realistic contexts.

The activities by stages

First stage

Examples for calculating areas and calculating the area of the surface and lateral surface of cylinders on the basis of given drawings

1. Calculating the volume and calculating the area of the lateral surface of cylinders whose data are indicated in Figure 1: In the first cylinder the base radius and the cylinder height are given and in the second the base radius and the ABCD area of the rectangle are given.

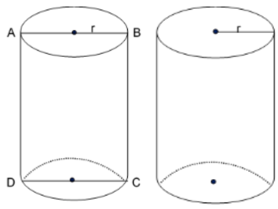


Figure 1

2. Drawing: different nets of solids of a cylinder. The drawing facilitates enhanced understanding and in-depth view of the cylinder components, i.e. a rectangle and two circles. This understanding enables better coping with computations of the volume and of the area of the lateral surface later on when tasks without illustrative drawings are given.
3. Connecting a cylinder volume to every day usage: a cylinder-shaped container is given as well as its base and height (e.g. 1000 cm² and 20cm) are given. The container is filled with four liters of water. Question: "What will be the height of the water surface after filling the container?" When solving the problem, the learners should translate the text into a 2-D drawing, add the numerical data and imagine the 3-D solid from everyday life.
4. Drawings of graphs. For example: a cylinder-shaped empty receptacle is given. It is being filled with liquids at a uniform and continuous pace. The learners have to draw a sketch of a graph which describes the relation between the filling time and the height of the liquid surface

Second stage

Integrating an activity with a cylinder and additional 2-D shape or 3-D solid.

1. The task described in Figure 2 displays four cylinders whose measures/sizes are different and the radius and height of each of the cylinders are known. Different triangles are inscribed within the cylinders. The learners have to explain and give arguments to the question in which of the surfaces the edge marked in 'bold' is the shortest or the longest. Moreover, they have to calculate the volume of the cylinder, the areas of the triangles and the length of the marked edge in each of the triangles.

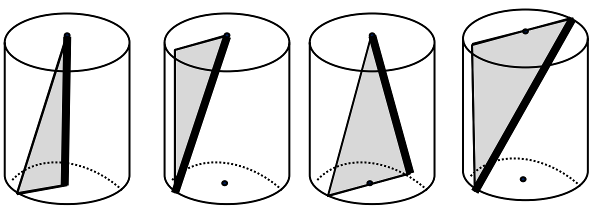


Figure 2

2. The task displayed in Figure 3 describes reciprocal situations between a cylinder and other solids. For example, a cylinder inscribed within a cuboid is given and the length of the base radius and the height of the cylinder are known. The volume of the cuboid should be calculated. Alternately, we have a squarecuboid inscribed within a cylinder whose height is given. ABCD is a square the measures of which are known. The learners have to calculate the volume of the cylinder.

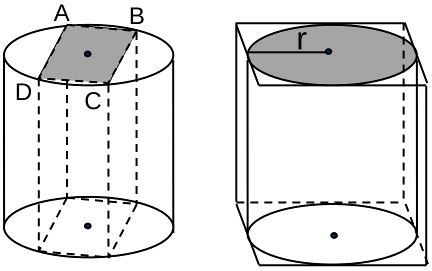


Figure 3

3. Drawing 4compares two cylinder-shaped receptacles. The story presented in the task forms the connection to reality. Tom wanted to buy a honey jar. The natural products shop to which he went sold honey in cylinder-shaped jars. He hesitated between two sizes of jars filled with honey, the one was tall and the other short.Examining the jars, he realisedthat one jar was twice as tall as the other but the radius of its base was twice as small.. The price of the tall jar was US$ 13.00 whereas the price of the short jar was US$ 20.00. Which jar should Tom choose if he wishes to buy the honey at the lowest price?

Figure 4

Third stage
From nets of solids to a solid

Figure 5

The task presented in drawing 5 engages in building a cylinder from a bristol sheet whose dimensions are 70x50 cm².

The learners should start by decomposing and assembling cylinders in order to become acquainted with the nets of cylinders even before coping with the current task. In order to build the cylinder from a bristol sheet, they have to draw the two circles of the cylinder base, the rectangular lateral surface and then cut the parts. Every Bristol sheet is given in a different colour for the purpose of distinction between them. It is recommended using adhesive strips in order to connect the parts and create the cylinder with them. Below are four different examples of nets of solids drawings.
Can a cylinder be built from each of these solids of nets? (Please note that the rectangle can be wrapped on the cylinder base more than once).
There are three main ways for checking whether a cylinder can in fact be formed:

1) A solution by tangible means, i.e. cutting and pasting.
2) A solution based on considerations relating to visual illustration.
3) A formal solution by computations.

A solution by tangible means, i.e. cutting and pasting

Drawing A –  the lateral surface does not suffice for wrapping the two circles and hence no cylinder can be created.

Drawing B –  the lateral surface wraps the two circles more than once and therefore a cylinder can be formed.

Drawing C –  the lateral surface is insufficient for wrapping the two circles and therefore **no** cylinder can be created.

Drawing D –  the lateral surface is insufficient for wrapping the two circles more than once and therefore **no** cylinder can be formed.

This activity illustrates that in fact the area of the bristol sheet is identical and in all of them there are two base circles and rectangular lateral surface. Nevertheless, not in all the cases a cylinder can be formed. The activity is tangible and every one can try creating a triangle in order to obtain the answers.

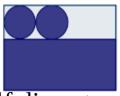
A solution based on considerations relating to visual illustration

These drawings are similar in that all of them have a rectangular lateral surface and two base circles. However, they are different in the dimensions of the lateral surface and size of the bases. The circumference of a circle equals twice the radius multiplied by π, namely multiplication of the diameter by π. Thus, in order to check whether the length of the rectangular lateral surface is sufficient for wrapping the base circle we should check whether the diameter of the circle is 'included' π times in the length of the lateral surface.

In drawing A –  the length of the rectangular lateral surface is exactly equal to the length of two diameters. Therefore no cylinder can be created.

In drawing B -  the length of the rectangular lateral surface is (even) bigger than the length of four diameters. Hence, a cylinder can be formed.

In drawing C –  the lateral surface is rectangular and we can see that its length is smaller than the length of three diameters. Consequently, no cylinder can be built.

In drawing D –  the length of the rectangular lateral surface is approximately equal to the length of three and a half diameters. Therefore, it is impossible to form a cylinder.

A solution in this way is based on comprehension and previous knowledge acquired about the meaning of the π.

A formal solution by computations

In drawing A –  the dimensions of the bristol sheet are 70x50. Hence, the radius length of each circle is: r=70:4=17.5. The circumference length of the one circle must be: $2\pi r \leq 70$. If r=17.5, we will obtain: $2\pi \times 17.5 \leq 70$. From this, $\pi \leq 2$ is derived. Net of solids A <u>does not enable building a cylinder.</u>

In drawing B –  the following equation is obtained: $2\pi r \leq 70$, $r \leq \frac{70}{2\pi} = 11.14$ – The circles in drawing B show that the length of two diameters is shorter than half the length of 70 cm, namely 4r<35 and therefore r<8.75. The conclusion drawn is that from net of solids B a cylinder can be built.

In drawing C –  should exist $2\pi r \leq 50$, $r \leq \frac{50}{2\pi} = 7.96$ –Nevertheless, we know for sure that r = 10 and the conclusion drawn is therefore that from net of solids C no cylinder can be formed.

In drawing D –  :$2\pi r \leq 70$ must exist. Hence, $r \leq 11.14$. The conclusion drawn is that no cylinder can be created from net of solids D.

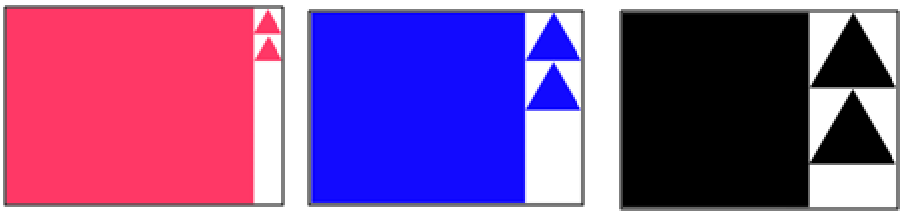
<u>A challenge question:</u> Is it possible to build an open cylinder (without a 'cover' = with one basis) from the above drawings?

The solutions based on considerations related to visual illustrations indicate that no numerical figures and complex computations are needed in order to solve the task. Learners as well as pre-service teachers and in-service teachers who approach the problem by means of demonstration aids manage to cope with it relatively easily. Learners who cope only with the tangible task – using a drawing but do not cut and paste in practice, try imagining and understanding what they have to do. However, most of them use the numerical figures, make computations, get confused and do not offer appropriate solutions for the task.

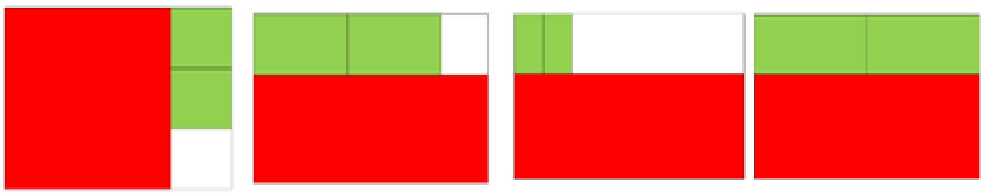
Expansion of the activity

Below are two additional examples which link and integrate a network of a solid and the building thereof:

a. The bristol sheet is 80x60 – please find in which drawings there is a possible network of a triangular prism.



b. Learners should try building a cuboid from a bristol sheet whose dimensions are 70 x 50 cm (please note that one can wrap the rectangle on the cuboid base more than once.



To sum up, in order to improve the spatial visualisation and ability which affect our daily life, it is important to present to the learners activities which apply computations and visual illustrations. These activities are suitable to learners of geometry in the higher grades of elementary school, pre-service mathematics teachers and teachers. Visualisation was applied in the activities presented above while integrating also a formal aspect.

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