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Relationship of Algebra I Teachers' Beliefs and Their Knowledge of Algebra for Teaching

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ABSTRACT: *This article presents a study of teachers' mathematical beliefs and Knowledge of Algebra for Teaching (KAT) of Algebra I teachers in Oklahoma. A Chi-Square Test for Independence was run on the sample of 144 teachers to determine if a high level of Knowledge of Algebra for Teaching score can predict a high problem-solving or constructivist view of algebra. A comparison of Algebra I teachers with a deeper background in mathematics content was compared to their beliefs as well. The study finds that those Algebra I teachers who have high KAT scores are dependent upon having a more problem-solving view of the nature of algebra. Additionally, the study indicates those Algebra I teachers with a high KAT are dependent upon having a deep mathematics content background. The results of this study suggest (1) those teachers in an Algebra I classroom are better prepared to teach algebra concepts with a problem-solving approach if having a greater KAT and (2) teachers with a deeper mathematics content background are more likely to have a high KAT.*
KEYWORDS: *Teaching mathematics, Algebra I, knowledge of algebra for teaching*

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I. INTRODUCTION

The teacher shortage crisis in the country is only predicted to get worse with the enrollment in teacher preparation programs decreasing over 35 percent from 2009 – 2014 (Aragon, 2016). Researchers (Ingersoll & Smith, 2003; Murnane et al., 1991) have suggested that due to this shortage, overall teacher quality is decreasing as schools continue to fill their classrooms with underqualified teachers, especially in the areas of mathematics and science.

Since teacher quality has been found to be a significant factor in student achievement, researchers examined the phenomenon of out-of-field teaching (Darling-Hammond, 2000; Ingersoll, 1999). Ingersoll (1999) focused on the definition of out-of-field teachers to be those "teachers assigned to teach subjects for which they have little training or education" (p. 26). Nearly one-third of secondary mathematics teachers do not hold a degree in mathematics or mathematics education (Ingersoll, 1999). The idea that out-of-field teachers' subject matter knowledge can affect teacher quality has been discredited by some researchers; however, research indicates that student achievement is significantly higher in Algebra I compared to other general mathematics courses when the teacher is fully certified in middle and secondary mathematics (Hawk, Coble, & Swanson, 1985).

These out-of-field teachers, from traditional and alternative certification pathways, may be underprepared to teach the mathematics course they are assigned. Student success and strong conceptual understanding of Algebra I content has shown implications related to performance in college, career readiness, impact on career salary, and student perception on higher mathematics (Eddy et al., 2015; Gaertner, Kim, DesJardins, & McClarty, 2014).

Since not all mathematics teachers are trained through the same certification pathway and may not share similar educational backgrounds, teachers' mathematical beliefs can affect how teachers conceptualize mathematics and its learning and teaching (Andrews, 2007; Cooney, Shealy, & Arvold, 1998). Teachers' beliefs about mathematics and its teaching are influenced by their own experiences in school (Borko et al., 1992) and those beliefs "play a significant role in shaping teachers' characteristic patterns of instructional behavior" (Thompson, 1992, p. 130).

Teacher quality and teacher characteristics in the classroom are found to be especially important in mathematics more so than in any other subject (Nye, Konstantopoulos, & Hedges, 2004). Teacher quality can be described in terms of certification and degrees held, but can also be described in terms of teacher practices in the classroom to further student achievement (Goe, 2007). Research on teacher quality based on qualifications such as certification pathway, level of education and subject matter knowledge differ across subjects (Betts, Zau, & Rice, 2003; Boyd, Grossman, Lankford, Loeb, & Wyckoff, 2006). When a teachers' deep subject matter

knowledge is combined with teacher education coursework, significant increases in student achievement in mathematics have been shown (Betts, Zau, & Rice, 2003; Clotfelter, Ladd, & Vigdor, 2006). Teacher characteristics including demographic (ethnicity and gender) and non-cognitive (beliefs and attitudes) factors affect teacher quality (Goe, 2007). Researchers have found mixed results identifying these types of teacher characteristics to have an impact on student achievement (Ehrenberg, Goldhaber, & Brewer, 1995).

This study uses a conceptual framework based on the work from Darling-Hammond (2000) and Ernest (1989) with those specific teacher characteristics that were found to be directly or indirectly related to student achievement through teacher practice. Trends have taken place in the categories of (a) subject matter knowledge, (b) knowledge of teaching and learning, (c) teacher beliefs, and (d) certification pathway (Darling-Hammond, 2000; Ernest, 1989).

Knowing subject matter and being able to use it is at the heart of teaching all students (Ball, 2000). During the 1980s, there was a plethora of research that showed a teachers' knowledge of a subject could affect their students' learning opportunities (Lampert, 1986; Leinhardt & Smith, 1985; Shulman, 1986; Wineburg & Wilson, 1988). Banks and Necco (1987) found that student achievement gains occurred specifically in algebra classes more often than general mathematics classes when the teacher held a degree in mathematics. Teachers should not just have high subject matter knowledge, but be able to "explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to other propositions" (Shulman, 1986, p. 9). Shulman (1986) defined this to be pedagogical content knowledge (PCK).

For over the last decade, mathematics education researchers studied teachers' combined subject matter knowledge and PCK to determine the high impact they both have for high quality teaching (Ball, 2003; Hill, Ball, & Schilling, 2008). The exploration of both subject matter knowledge and PCK brought about the term of content knowledge for teaching (MKT), explained to be the following: (1) Knowing the content that the students are supposed to learn; (2) Knowing ways to unpack, represent, and make that content learnable; (3) Knowing how students think about the specific content; (4) Knowing ways to teach the specific content (Ball, Thames, & Phelps, 2008, p. 3).

MKT of secondary mathematics teachers is a focus for researchers, specifically in the area of algebra (Li, 2011; McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012). McCrory et al. (2012) found that secondary teachers' knowledge of teaching is strictly measured through content tests or by the number of mathematics courses on their transcripts, where neither of these measures given are a certain to produce effective teaching of algebra. Research has shown that a strong MKT is not always guaranteed by taking a certain number of higher-level mathematics courses (Even, 1999). Studies have linked high levels of student achievement with teachers who attend professional development where content-based pedagogy is being taught (Brown, Smith, & Stein, 1996; Cohen & Hill, 2000; Wiley & Yoon, 1995).

Beliefs are referred to as "psychologically held understandings, premises, or propositions about the world that are thought to be true" (Richardson, 1996, p. 259). Research on teacher beliefs found beliefs to be highly individualized for every teacher, greatly influential in teaching practices, and impactful on student achievement (Lepik & Pipere, 2011; Love & Kruger, 2005). The three main areas of teacher beliefs are discussed in here including beliefs about nature of mathematics, learning of mathematics, and teaching of mathematics.

Teacher beliefs about nature of mathematics. Historically, philosophical views about the nature of mathematics have fallen on two extremes. On one extreme, mathematics is viewed as "static, fixed, and either discovered or waiting to be discovered" (Amirali & Halai, 2010, p. 47) while the other extreme view of mathematics is "seen and interpreted as socially constructed phenomena" (Amirali & Halai, 2010, p. 47). Furthermore, Ernest (1989) suggests that views on the nature of mathematics typically fall into one of the following three views: (1) Dynamic problem-solving view - Mathematics is a continually expanding field of human inquiry where it is an unfinished product and its results remain open to revision. (2) Platonist view - Mathematics is a static, but unified body of knowledge, consisting of truths which are discovered, not created. (3) Static instrumentalist view - Mathematics is a useful, but unrelated collection of facts, rules, and skills (p. 21).

Furthermore, Ernest (1991) states that teachers' nature of mathematics views can have implications on their teaching practices. Teachers with a dynamic problem-solving view of mathematics use a non-directive and open-teaching style (Lerman, 1990). Others with a Platonist view of mathematics use related rules and facts attempting to find links between concepts. Teachers with a static instrumentalist view consider themselves an authority in the classroom and their role is to impart mathematical knowledge to their students (Ernest, 1991). Francis (2014) found that instruction of teachers holding a dynamic problem-solving view of the nature of mathematics brought more understanding to students and more desire to learn mathematics.

Teacher beliefs about learning mathematics. Studies on teacher beliefs about learning mathematics revealed that a majority of teachers believe that learning mathematics is most effective in a traditional lecture-style method (Peterson, Carpenter, Fennema, & Loef, 1989). This is in contrast to mathematics educators who believe the role of a mathematics teacher "is to guide and support students' invention of viable mathematical

ideas rather than transmit 'correct' adult ways" (Clements & Battista, 1990, p. 35). Civil (1990) revealed that mathematics teachers emphasized neatness and speed as the best way to solve a problem in the classroom. Other researchers have found that many mathematics teachers perceive mathematics learning best occurs when it is based on memorization of rules and algorithms (Southwell & Khamis 1992).

Teacher beliefs about teaching mathematics. Research has shown that teacher beliefs about the teaching of mathematics is similar to the traditional beliefs of learning mathematics where teaching is about providing step-by step procedures and then students mimicking those procedures (Stipek et al., 2001). In a study of 249 secondary mathematics teachers, Howard, Perry, and Lindsay (1997) found teacher beliefs tended to fall into two types of beliefs about teaching that they called transmission and constructivist. They found that the majority of the teachers fell into the transmission view of teaching mathematics where they believed that mathematics is based on memorization of rules and procedures, while the remaining few teachers held a constructivist view of teaching mathematics where students were encouraged to explore, propose, and explain solutions.

Similarly, Van Zoest, Jones, and Thornton (1994) studied two groups of pre-service mathematics teachers where one group was enrolled in a mentorship program based on a philosophy of constructivist teaching and the other group served as the control group. Results of this study revealed that regardless of completing the mentorship program, both groups had resorted to a more traditional set of beliefs about teaching mathematics. These traditional sets of beliefs about teaching mathematics are so engraved in teachers' beliefs that reforming those beliefs has proven to be difficult (Perry, Tracey, & Howard, 1999).

This study aims to examine any relationship between teachers' algebra beliefs and their Knowledge of Algebra for Teaching (KAT). Additionally, it examines whether teachers' subject matter knowledge in mathematics has a relationship to their algebra beliefs and their KAT. Thus, the research questions that guided this study were: (1) Is there an association between algebra teachers' Knowledge of Algebra Teaching (KAT) and their beliefs about algebra, about teaching algebra, and about learning algebra? (2) Is there an association between an Algebra I teachers' content knowledge of mathematics and their beliefs about algebra, about teaching algebra, and about learning algebra? (3) Is there an association between an Algebra I teachers' content knowledge of mathematics and their Knowledge of Algebra for Teaching (KAT)?

II. METHODOLOGY

An online questionnaire was sent to all Oklahoma public school mathematics teachers, where only teachers currently teaching Algebra I during the 2016 – 2017 academic year completed the questionnaire. The Oklahoma State Department of Education divided the state into eight geographic regions called the REAC^{3}H regions where all of the 516 school districts are assigned. The questionnaire was completed by 144 Algebra I teachers from across the eight regions, which resulted in a 5.4 percent response rate and was representative of the state (see Table 1). There was almost an even distribution of teachers with a Bachelor's degree (49.3%, n = 71) and teachers with a Master's degree (50%, n = 72). Teachers with 10 or more years of teaching experience comprised about 60 percent of the sample while those with 0 – 5 years or 6 – 10 years of teaching experience each comprised 20 percent.

Table 1: Representativeness of Teachers in Oklahoma Reac^{3}h Regions[1]

	Population		Sample	
Reac^{3}h Region	Number	Percentage	Number	Percentage
1	113	4.23	6	4.17
2	191	7.15	20	13.89
3	550	20.60	26	18.06
4	341	12.73	11	7.63
5	185	6.93	9	6.25
6	229	8.58	17	11.81
7	205	7.68	8	5.56
8	856	32.06	47	32.64

Note. [1] The Oklahoma Reac^{3}h regions were used to determine the geographical representation of the state. A map of the Reac^{3}h regions can be found at http://ok.gov/sde/reac3h-network.

The participants completed three different instruments through an online questionnaire. Participants were asked to provide demographic information, respond openly about their beliefs about algebra, and participate in a 20-question assessment that measure their Knowledge of Algebra for Teaching (KAT). The last three questions on the KAT were open ended and the process of uploading solutions was time-consuming, which resulted in teachers not completing these. Thus, the range of scores on the KAT were adjusted to not include those last three open-ended questions so that teachers who did not complete that portion were still included in the study.

The first questions on the questionnaire required the Algebra I teacher to declare the current grade(s) they were teaching, school name, and the district. This allowed the researcher to classify each teacher into the appropriate REAC^{3}H region to check representativeness for the state of Oklahoma. The next demographic information collected included the teacher's years of experience teaching mathematics, Bachelor's and/or Master's degrees achieved, and pathway to certification.

The algebra beliefs questionnaire used in this study is a modification of Raymond's (1997) beliefs questionnaire where all mentions of "mathematics" were changed to "algebra." Although the current instrument has not yet been validated, it was thoroughly examined by two mathematics educators at Oklahoma State University to certify the questions were appropriately measuring teacher beliefs. The beliefs questionnaire contains three subscales – beliefs about the nature of algebra, beliefs about learning algebra, and beliefs about teaching algebra. The Cronbach's alphas for each of the three subscales in this study were calculated as .81, .75, and .54, respectively, using the data from this study. Each of the three subscales have a series of semantic differential questions and a group of 5-point Likert-type questions. The beliefs about the nature of algebra subscale has a possible score range of 16 to 176 containing 8 of each type of question. For the beliefs about learning algebra and teaching algebra, the possible range of scores are 17 to 187 (7 semantic differential questions and 10 Likert-type questions) and 15 to 165 (8 semantic differential and 7 Likert-type questions), respectively. The higher score on the beliefs about learning and teaching of algebra portion, the more constructivist view of algebra is suggested (Clements & Battista, 1990).

The Survey of Knowledge of Algebra for Teaching measures the knowledge that is most effective in teaching algebra – teaching knowledge specific to algebra concepts (Tscore), knowledge of middle and high school algebra (Sscore), and advanced knowledge of mathematics typically taught in college level mathematics (Ascore). A final score is given that incorporates all three dimensions of the KAT simultaneously. The instrument had a Cronbach alpha reliability of .84 during the validation study. The Cronbach alpha for this study was .73. Out of the 17 questions on the KAT, five of the questions cover Tscore, eight cover Sscore, and four include topics in Ascore. The range of scores for the final score and each of the dimensions when using all 20 questions and the adjusted scores when the three open-ended questions were removed.

Results were analyzed using inferential statistics with SPSS (Version 18.0). Inferential statistics included the use of a Chi-Square Test of Independence with cross tabulation. The test calculated whether there was an association between Algebra I teachers' KAT score level and their beliefs about algebra by comparing the observed outcomes to the expected outcomes. The test also calculated whether there was an association between an Algebra I teachers' mathematical content background and their beliefs about algebra. All assumptions for the Chi-Square Test of Independence were ensured including checking that all the expected cell counts were greater than five. Scatterplots were used to visualize data and make comparisons.

III. DISCUSSION

The purpose of this study was to (a) explore the association between an Algebra I teachers' KAT and their beliefs about the nature of algebra, learning of algebra, and teaching of algebra and (b) explore the association between an Algebra I teachers' KAT and their mathematics content background. This research is significant because it is important to know Algebra I teachers' beliefs and MKT since they both can have an impact on instruction.

Findings in this study suggest that Algebra I teachers in the state of Oklahoma hold a problem-solving view of mathematics when they have a higher KAT. More than three-fourths (81%) of the teachers with a deep subject matter knowledge (mathematics education and mathematics pathways) tended to hold this dynamic problem-solving view and held high KAT. This finding coincides with previous research that found when teachers have taken more mathematics content courses along with teaching methods courses there was an increase in their MKT (Hill et al., 2008; Hill, Rowan, Ball, 2005; Darling-Hammond, 2000). Additionally, Francis (2014) suggested that teachers holding beliefs that support problem-solving views of mathematics is considered one of the most valuable factors affecting teacher quality. Kim (2005) noted that when teachers have a dynamic problem-solving view of mathematics (a) they tend to create a classroom environment based on constructivist teaching practices and (b) their students' achievement in mathematics has been associated with higher gains, particularly in algebra. Teachers who hold a dynamic problem-solving view tend to approach problems in multiple ways and have expectations for their students to solve problems in a variety of ways (Amirali & Halai, 2010).

In terms of the mathematics background, having a mathematics degree seems to have a large effect on the KAT of Algebra I teachers, but not on their beliefs. Teacher certification pathway has shown a significantly positive effect on mathematics achievement of students when the teacher is certified in mathematics education (Rice, 2003). Additionally, previous studies (Corkin, Ekmecki, & Fan, 2016; Hill et al., 2005) have found that teacher coursework in the specific content area and pedagogy yields a higher quality of teachers in the classroom. These results suggest that many of the Algebra I teachers in the state possess a low knowledge of

algebra and its teaching. This calls into question the depth and rigor of the subject area test that allows these teachers to teach Algebra I and above. Additionally, with the high number of elementary certified teachers teaching Algebra I, how exposed are they to the tools and methods to effectively teaching Algebra I concepts before entering the classroom?

An implication of this study includes ensuring those teachers in an Algebra I classroom have a high Knowledge of Algebra for Teaching (KAT) for the most effective teaching to take place in the classroom. Although the beliefs about the nature of algebra, learning of algebra, and teaching of algebra are known to be difficult to change (Gill, Ashton, & Algina, 2004), attention still needs to be paid to them. Along with increasing KAT, professional development for mathematics teachers should focus on forming those beliefs about problem-solving and constructivist views. Due to the effect that a teachers' mathematics content background has on their KAT, it is advised that more rigorous standards for certification pathways be explored to enhance the student achievement in Algebra I. Additionally, for those teachers already holding a mathematics certification in the state, but who do not hold a sufficient mathematics educational background should be required to continue to deepen their content knowledge. This can take place through targeted professional development and/or Master's coursework rich in content and pedagogy related to algebra concepts. The enactment of these implications would only increase the teacher quality of Algebra I teachers in the state, and mathematics teachers in general, which ultimately would impact student achievement.

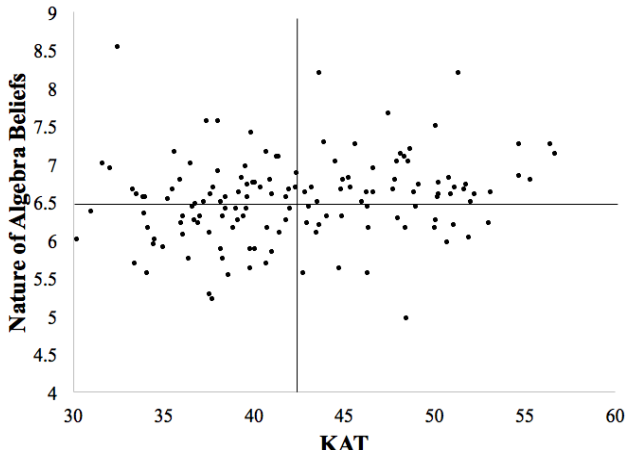
IV. FINDINGS

In order to explore any association between beliefs and KAT of Algebra I teachers, scatterplots and a Chi-square test of independence was run on contingency tables of high and low beliefs scores and KAT scores of those teachers. The Algebra I teachers' beliefs scores and KAT scores were classified into high and low categories depending on the scores falling above or below the overall means of the 144 Algebra I teachers in the sample. The overall means were – KAT final score (M = 42.35), beliefs about the nature of algebra (M = 6.50), beliefs about the learning of algebra (M = 6.21), and beliefs about the teaching of algebra (M = 5.39).

First, a visual representation of scatterplots was used to show the number of teachers from the sample that fell into each high-low category of KAT final score and each type of belief. The scatterplots in Figure 1 are based on the belief about the nature of algebra, learning of algebra, and teaching of algebra. Each scatterplot, regardless of type of belief, have the same KAT final score mean (M = 42.35) for the x-axis separating the high and low teachers. The overall mean scores for each belief, listed above, represent the y-axis separating the high and low teachers.

The scatterplot of teachers' KAT and nature of algebra beliefs indicate about 28 percent of teachers falling into the high KAT and high nature of algebra beliefs category, suggesting that these teachers have a problem-solving view of mathematics. A fairly equal percentage (30%) were found in the low KAT and low nature of algebra beliefs category. This suggests that these teachers have a more instrumentalist view of mathematics. With more than half of the sample falling in one of these categories, the trend of this scatterplot may suggest an association between KAT and nature of algebra beliefs. The percentage of teachers falling into high KAT and high learning of algebra category compared to those teachers falling into the high KAT and high teaching of algebra were 19% and 23%, respectively. This suggests these teachers have a more constructivist view of learning and teaching mathematics. No trends appeared to take place in the learning and teaching of algebra scatterplots due to the data being fairly evenly distributed in each high-low category.

Figure 1
Sample of Algebra I Teachers



To confirm the visual inspection of the data, a Chi-square test of independence was performed. The Chi-square test of independence indicated significant association between an Algebra I teachers' KAT final score and nature of algebra beliefs, 2(1, n = 144) = 5.76, p = .016, phi = .2. These results suggest that those Algebra I teachers with a high KAT final score are more likely to have a high nature of algebra beliefs score, indicating a more dynamic problem-solving view of algebra. The results also suggest the same for those Algebra I teachers who have a low KAT final score more likely to have a static instrumentalist view about the nature of algebra. The effect size of this association is considered to be a small to medium effect (Cohen, 1988). A Chi-square test of independence showed no association between an Algebra I teachers' KAT final score and belief about the learning of algebra, 2(1, n = 144) = .36, p = .547, phi = .05 and belief about the teaching of algebra, 2(1, n = 144) = 2.07, p = .150, phi = .12. Results indicate that those teachers who have a high KAT do not necessarily have a relationship to having a constructivist view of learning and teaching of algebra. Additional analyses were run on the three sub-dimensions of the KAT and three beliefs about algebra. Table 2 shows there was no significant dependency between any of the three KAT dimensions to any of the three types of beliefs about algebra.

Table 2: Cross Tabulation of Mathematics Background with Beliefs and KAT

	Mathematics Degree			
	No	Yes	2	
Nature of Algebra				
Low	32	36	1.16	.09
High	29	47		
Learning of Algebra				
Low	30	49	1.38	.10
High	31	30		
Teaching of Algebra				
Low	27	40	.22	.04
High	34	43		
KAT Final Score				
Low	49	30	27.72**	.44
High	12	53		
KAT Tscore				
Low	47	38	14.42**	.31
High	14	45		
KAT Sscore				
Low	52	37	24.63**	.41
High	9	46		
KAT Ascore				
Low	48	31	24.26**	.41
High	13	52		

Note. $^{**}p < .001$.

In order to explore any associations between an Algebra I teachers' level of subject matter knowledge and their beliefs of algebra, the sample of Algebra I teachers was broken down into two levels of mathematical content knowledge based on educational background as follows: (1) Mathematics Degree (n = 83) – any teacher who obtained a Bachelor's degree in mathematics or the equivalent. (2) No Mathematics Degree (n = 61) – any teacher who did not obtain a Bachelor's degree in mathematics or the equivalent.

A Chi-square test of independence indicated significant association between an Algebra I teachers' KAT final score and their mathematical content knowledge, 2(1, n = 144) = 27.72, $p < .0001$, *phi* = .44. These results suggest that those Algebra I teachers who hold a mathematics degree are more likely to have a high KAT. The results also indicate the same for those Algebra I teachers who do not hold a mathematics degree are more likely to have a low KAT. The effect size of this association is considered to a medium to large effect (Cohen, 1988).

The Chi-square test of independence showed a significant association between Algebra I teachers' mathematical content knowledge and every dimension of KAT, including Tscore ($2(1, n = 144) = 14.42$, $p < .001$, *phi* = .31), Sscore ($2(1, n = 144) = 24.63$, $p < .001$, *phi* = .41), and Ascore ($2(1, n = 144) = 24.26$, $p < .001$, *phi* = .41). The effect size of every test is considered to be a medium to large effect (Cohen, 1988). The results indicate that a higher teaching knowledge of algebra (Tscore), knowledge of middle and high school algebra concepts (Sscore), and advanced knowledge of mathematics (Ascore) is strongly associated with having a degree in mathematics.

A Chi-square test of independence indicated no association between an Algebra I teachers' mathematics content background and belief about the nature of algebra, $2(1, n = 144) = 1.16$, $p = .28$, *phi* = .09, belief about the learning of algebra, $2(1, n = 144) = 1.38$, $p = .24$, *phi* = .10, and beliefs about the teaching of algebra, $2(1, n = 144) = .22$, $p = .64$, *phi* = .04. Results indicate that those teachers with who hold a mathematics degree do not show a relationship to having a dynamic problem-solving or constructivist view of algebra.

V. CONCLUSION

With Algebra I being the gatekeeper course for higher mathematics achievement along with college and career readiness, questions about the quality of teachers in those classrooms arise (Stoelinga & Lynn, 2013). Research has shown that the factors such as subject matter knowledge, knowledge of teaching and learning, and certification pathway can influence teacher practices in the classroom and affect student achievement (Darling-Hammond, 2000; Goldhaber & Brewer, 2000; Hill, Rowan, & Ball, 2005). Teachers' mathematical beliefs are highly individualized with many teachers falling back into the traditionalist view of mathematics regardless of their certification pathway or educational background (Gudmundsdottir & Shulman, 1987; Perry, Tracey, & Howard, 1999). Since Algebra I can play such a key role in the future mathematical success of students, is close enough attention given to those teachers' qualities and beliefs before entering those classrooms?

The research in this study explored those specific Algebra I characteristics and teacher qualities in the state of Oklahoma by measuring their Knowledge of Algebra for Teaching (KAT), beliefs about the nature of algebra, learning of algebra, and teaching of algebra. Additionally, comparisons of those measurement were made across different certification pathways and mathematical backgrounds.

In this study, quantitative data was collected from 144 teachers currently teaching Algebra I during the 2016 – 2017 academic year in the state of Oklahoma. Data about the teachers were collected through an online questionnaire using demographics questions, an algebra beliefs questionnaire, and the Survey of Knowledge of Algebra for Teaching (KAT) instrument. Results were used to portray a picture of Algebra I teachers in Oklahoma using demographics, certification pathways taken, and mathematics background. The quantitative results from the algebra beliefs questionnaire and KAT were analyzed to find any differences or associations between those teachers' beliefs, KAT, certification pathways, and mathematics backgrounds. The results of the study were organized into two articles, which are summarized in the following pages.

Overall, there were three main findings from this study. The first finding indicates that teachers' algebra beliefs in Oklahoma do not differ regardless of certification pathway, but teachers' KAT level are dependent upon certification pathway and mathematics content background. The second finding indicates that Oklahoma Algebra I teachers with a higher KAT are associated with having a more problem-solving of the nature of algebra, but not with having a constructivist view of the teaching and learning of algebra. The third finding indicates those Algebra I teachers certified through a traditional mathematics education program consistently have a higher KAT, higher problem-solving view of mathematics, and higher constructivist view of teaching and learning algebra than those certified through any other pathway. The focus and significant findings from each chapter article are discussed in the rest of this section.

The main teacher characteristics of Algebra I teachers in Oklahoma were examined by age, teaching experience, educational background, and certification pathway. While nearly 20 percent of teachers have only been teaching mathematics for 1 – 5 years, the average age of Algebra I teachers in Oklahoma is 43 years old. With exactly 50 percent of teachers holding a Master's degree, it is important to point out that 32 percent of them hold a Master's degree in mathematics education. The majority of certification pathways taken in the state are through a traditional teacher preparation pathway with 47 percent certified through a mathematics education pathway and 16 percent certified through an elementary education pathway. An additional 11 percent of teachers hold a Bachelor's degree in mathematics, but were certified alternatively.

Overall, the teachers' beliefs about the nature of algebra were consistently higher, meaning a higher problem-solving view of algebra, than those beliefs about learning and teaching algebra. Similarly, data revealed, that regardless of certification pathway, teachers held similar beliefs about the nature of algebra, learning of algebra, and teaching of algebra. These results suggest that even those teachers certified through a traditional teacher education program may fall back more into an instrumentalist view of mathematics and a non-constructivist view of teaching and learning mathematics. The findings of Raymond (2007) and Prawat

(1992) suggested the same with teachers' prior school experience being the main influence on teacher beliefs pushing more traditional and procedural teaching techniques.

Findings in this study indicated certification pathway to have a strong effect on the content knowledge and teaching knowledge of Algebra I teachers. Teachers certified through a mathematics education pathway and those certified through a mathematics alternative pathway consistently had higher algebraic content knowledge and teaching algebra knowledge. Previous research (McCrory et al., 2012) points out the importance of these findings suggesting teachers at the middle and secondary level need to have a deep knowledge of advanced mathematics in order to effectively teacher mathematics. Additional previous research showing effective classroom instruction being strongest when a high subject content knowledge and pedagogical knowledge are held points out the importance of these findings (Ball, Thames, & Phelps, 2008; Shulman, 1986). This research may also suggest elementary education certified teachers and other non-mathematics based majors do not have a deep enough mathematical content background to effectively teach algebra.

The findings in this study also suggested that Algebra I teachers who have a higher overall KAT tend to have higher nature of mathematics views, meaning a more problem-solving view. This association was mainly comprised (81%) of teachers certified through a mathematics education pathway and those certified through a mathematics alternative certification pathway. This suggests those teachers holding a mathematics degree are most likely to have a dynamic problem-solving view of mathematics. Previous research found similar results of teachers with more mathematics content courses combined with mathematics methods courses, there was an increase in their MKT (Hill et al., 2008; Hill, Rowan, & Ball, 2005; Darling-Hammond, 2000). Furthermore, this aligns with research suggesting a dynamic problem-solving view of mathematics to be one of the most valuable teacher qualities in mathematics by approaching problems in multiple ways and pushing for students to solve problems in a variety of ways (Amirali & Halai, 2010; Francis, 2014).

Additionally, results indicate that having a deep mathematics content background has a strong association with all dimensions of KAT, including teaching knowledge, school algebra knowledge, and advanced mathematical knowledge, but not on their beliefs. Similar research findings suggest the same (Corkin, Ekmecki, & Fan, 2016; Hill, Rowan, & Ball, 2005; Rice, 2003). These results indicate there are many teachers in the state who hold a low content knowledge of algebra and its teaching since there are a high number of elementary certified teachers in an Algebra I classroom.

The results of this study showed that a teachers' mathematics content background did not have an association with the teachers' beliefs about the learning and teaching of algebra. These results suggest that regardless of mathematics content background, teacher beliefs are most likely influenced by their own previous experiences in a mathematics classroom, when most teachers were taught using a traditional teaching style. These findings align with previous research by Raymond (1997).

Implications

The results from these two studies have implications for Algebra I teachers. First, the requirements to become certified to teach in an Algebra I classroom should be more strict and rigorous. Middle and secondary mathematics teachers need to understand the connection within different branches of mathematics and convey importance of the learning of mathematics (Usiskin, 2001). Teachers who do not follow a traditional mathematics education certification pathway are asked to just pass a mathematics content knowledge exam, but is this enough in order to ensure this effective teaching necessary in an Algebra I classroom? Since Algebra I is the gatekeeper to higher level mathematic, schools need to make sure high teacher quality in these classrooms is put first. There is a need for the state of Oklahoma to truly examine who is in Algebra I classrooms and the pathways those teachers can gain certification in order to teach this course.

Second, the most effective teaching in an Algebra I classroom takes place when the teacher has a high KAT. Ensuring that all teachers of Algebra I in Oklahoma have a high KAT is a necessity. Although the beliefs about the nature of algebra, learning of algebra, and teaching of algebra are known to be difficult to change (Gill, Ashton, & Algina, 2004), attention still needs to be paid to them. Professional development for teachers should always focus on increasing KAT, but a higher emphasis on forming beliefs to fit more problem-solving and constructivist views should be put in place. With the study showing such strong associations with mathematics content background to high levels of KAT, it is advisable that the state of Oklahoma implement more rigorous standards throughout the certification process to enhance student achievement in Algebra I. For those teachers who do not have a deep content mathematics background, but already hold a mathematics certification in the state of Oklahoma, steps to deepen their content knowledge should occur. Teachers can achieve this deeper level of understanding mathematics content by attending professional development or complete Master's coursework focused on content pedagogy specific to algebra concepts. These implications would increase the Algebra I teacher quality in Oklahoma and would help students achieve a higher level of understanding algebra concepts to carry with them into higher level mathematics courses.

Algebra I teachers should be of the highest teacher quality based on their beliefs, MKT, and mathematical content background. With the shortage of mathematics teachers, teacher quality is becoming more difficult than ever to maintain in mathematics classrooms (Aragon, 2016). This research study provides

evidence that Algebra I teachers mathematical background and certification pathway can highly develop a teachers' KAT. However, the teachers' beliefs about algebra will most likely be similar regardless of mathematical background or certification pathway. More rigorous processes need to be setup in order to ensure that Algebra I teachers come into the classroom ready to teach algebra concepts multiple ways and bridge the concepts to other mathematics courses. Unless the KAT of teachers continues to increase and the beliefs of Algebra I teachers move more toward a problem-solving and constructivist views, teachers may not have the strong understanding of algebra in to teach for the understanding of all students (NCTM, 2000). Without the high level of KAT, problem-solving views, and constructivist views, student achievement may be hindered.

Recommendations for Future Research

While the data from this study on Algebra I teachers added to the body of literature on beliefs, KAT, certification pathway, and mathematics background, further research is still needed on how to improve the teacher quality of Algebra I teachers. The following recommendations for future research from this study lead to these next studies: (1) It is known that classroom practice can be affected by mathematical beliefs of teachers. Future research should examine key factors that cause resistance to the changing of teachers' beliefs. (2) Although a challenge, researchers should explore the option of validating an instrument that measures teachers' beliefs about the nature of algebra more specifically according to the instrumentalist, Platonist, and problem-solving continuum. (3) Some participants in this study held a high overall KAT, a high KAT in every dimension, problem-solving view of algebra, and constructivists views of teaching and learning algebra. Interviews with those teachers would be interesting to determine if their practices in the classroom match their beliefs.

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