## Conceptions and Reasonings of Beninese Learners in Solving Physics Problems Involving the Object "Acceleration" in the Concept of "Rapidity"

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ABSTRACT: In this work, we describe the modes of reasoning of the learners in order to resolve two types of problems and involving the object "acceleration" in a scientific context and the object "rapidity" in an empirical context or everyday life. These learners are young Beninese students with a scientific G.C.E.A.Level and who have been trained according to the competency approach by qualified teachers with at least five years of experience and with course materials authorized by the inspection directorate education and secondary education. From the different analyzes (lexical and sequential), it emerges that in the resolution of these problems, the concepts, rules and formulas mobilized by these learners appear as concepts and theorems-in-act (Vergnaud, 1994) Categorized profiles and according to the contexts of investigation. Different modes of reasoning stem from their distances from the design intended for the object of investigation. If, according to Rey, Defrance and Kahn (2006) and Carette (2009), reasonings and conceptions are indispensable to be competent, it seems to us that the didactic choices prescribed by the actors of the Beninese education system in the implementation of this new approach will allow Difficult to achieve the objectives of the change of approach.

Keywords: Concept, situation, concept-in-act, theorem-in-act, open problem, school problem, reasoning.

### I. INTRODUCTION

Since 1960, the education system in Benin has seen the emergence of its curricula. The socio-political crisis of 1989 led Benin in February 1990 to the conference of the Living Forces of the Nation. One of the resolutions resulting from this conference was the organization of the General States of Education EGE (February 1990) which plunged the actors of the education system into a vast field of didactic reflection centered on the theoretical foundations of the Competence Approach (APC). For its implementation, a framework document has been drawn up and has favored all scientific disciplines and therefore also the Physical, Chemical and Technology Sciences (SPCT). This framework document has defined for the teaching of Development and acquisition of three types of competencies (disciplinary, transdisciplinary and transversal) through the Learning Situation (SA) and the Teaching / Learning (E / A) strategy (Documents Guide and Program terminal C, 2011). This methodological approach mainly focuses on a certain number of capacities to be developed in the learner, in particular on his conception in relation to an object of study and his mode of reasoning in problem solving, and thus justifies the option d Entry into the curriculum through competencies. According to Klein (2003) & Kahn (2006), the learning processes and techniques mobilized by the learner influence the construction of knowledge and determine its competence. Speaking of skills, Closset (1992) argues that reasoning is a necessary step to be competent. In the same vein, he argues that the students' reasoning is qualitatively the same but quantitatively different. According to Giordan (1996), the conceptions of a subject in relation to an object of knowledge represent a stepping stone for learning and therefore for being competent. For Roegiers (2000), the APC allows the construction of disciplinary capacities that are not very evaluable. He uses situations in everyday life to trigger the learning of a scientific concept. Our analysis focuses on the impact of the implementation of this introduced and widespread approach (from the sixth to the final year) in Benin for almost a decade. In the context of Benin, it seems that the implementation of this new approach makes it difficult to develop scientific and coherent reasoning among learners of the same level of study and to change their conceptions in relation to the objects studied. In this presentation, we analyze the productions of young students (learners) in the solution of problems involving knowledge of the class of scientific terminals in Benin.

### 1. Problematical

Since the advent of the New Study Programs (NPE) in Benin, several research studies have focused on the impact of this approach. Some works, for example, have shown the existence of several difficulties in the preparation and conduct of a class situation (Oké, 2012) and (Ayigbèdè, 2016). Others stressed the factors that influence the implementation of APCs, the stability and transversality of the skills acquired by the students and the learning and problematization (Agbodjogbé, 2011), (Sassou and Hounkpè, 2004). ). This work, like that of Vergnaud (1990-1994) on the learning of knowledge in mathematics, is an evaluation of the learning of a concept in the physical sciences and mobilizes notions of conceptions, concepts-in-act and theorems -act in the theory of conceptual fields. This work is based on that of Dumas-Carre A, Goffard (1992) on the difficulties of pupils related to the various problem-solving activities and uses the notions of "open problem" and "academic problem" (JM Boilevin, 2001) In teaching / learning and assessment of knowledge.

### 1.1. Concepts and reasoning in the learning of concepts

A conception thus designates a set of determinants characteristic of knowledge or principally of the understanding that a subject holds of an object in an institution (Vergnaud, 1990). This understanding beyond the class situation, can reveal itself through its reasoning in ordinary situations of life. A conception thus integrates primitive knowledge, the errors and misunderstandings that follow, the way in which they change or can change, the situation through which they are expressed, the explanations they mobilize and the steps they take (Vergnaud, 1990). In the logic of the two contexts defined by Robardet and Guillot J.C (1997), to solve a school problem, the learner mobilizes a scientific conception (CS) of the object of investigation. In this case, he uses a formula, a law, or an experimental field theorem to establish a mathematical solution. To solve an open problem without data, the learner uses an empirical (CE) design of the object of investigation. In the case it calls upon the concepts defining the object in the empirical context to establish the solution. To solve an open problem, the learner must identify data of the situation by matching the concepts and theorems of his theoretical and empirical registers. This reasoning is built on three levels:

- ✓ **Mathematical solution**: relation between the data of the statement to define the object the scientific conception of the object involved in the academic problem (1st level of correspondence);
- ✓ **Real solution**: relationship between the terms and concepts of everyday life defining each empirical design of the object involved in the open problem. (2nd level of correspondence);
- ✓ **Complete solution**: in which the learners succeed in establishing a correspondence between the linguistic signifiers of the two levels, between the relations established on the basis of the correspondences between the concepts and the theorems-in-act of the first two levels. (3rd level of correspondence).

### 1.2. Open and academic problems: Sources of conceptions and reasoning

Boilevin, J.-M. and Dumas-Carré, A. (2001), in the search for a physics problem solving activity model in initial teacher training have made conceptual clarifications on the problem. For these authors, a problem differs from the classical exercises by its different functions (learning and evaluation). It is an assessment tool because it targets the mobilization of skills and abilities. It also makes it possible to develop a reasoning based on knowledge and techniques. It necessarily takes into account a reality or a phenomenon of life and makes it possible to analyze how the transfer of scientific knowledge takes place in the situations of everyday life. It is more suited to the socioconstructivist method of teaching, learning and evaluating knowledge to acquire skills. A problem involves a task and includes one or more of the instructions that involve resolution procedures. In the light of these conceptual clarifications and the objectives of our study, we believe that a problem is better adapted to understand the learners' conceptions of an object of study and to describe the modes of reasoning elaborated with these conceptions. In reality, the problem imposes learners by their characteristics and functions, a production argued by knowledge and techniques.

### 1.3. Concepts and theorems-in-act in reasoning

A concept is defined both by a set of three components in which it regains its full meaning (Vergnaud, 1990). All three components work in close relation and each contribute to the concept and can be summarized by the scheme of Vergnaud (1990) below.

# The set of invariants on which rests the operationality of schemes (the signified). The set of situations that give meaning to the concept, question it or use it (the reference) The set of linguistic and non-linguistic forms that allow symbolic representation of the concept

Diagram 1: Showing the various components that characterize the concept in the sense of Vergnaud (1990). Among the three components defining the concept, the operative invariants (the signified) on which the organization of the activity of resolution of the situation is based represent one of the three components defining the concept which is determinant in the reasoning of the learners. They constitute a "property" or a "relationship" that is retained on a certain set of transformation". They bring together: Concepts-in-act concepts that are only relevant to information and are not likely to be truthful or false; The-in-act theorems that represent propositions held for "true" in activity and whose constructions are animated by concepts-in-act and which are susceptible of truth or falsehood within a Certain area. Particularly in problem solving, the operative invariants of the theorems-in-act and concepts-in-act are characteristic components on which the reasoning of a learner is based the resolution of problems involving knowledge.

### 1.5. Objective, research question and hypothesis

This research presents several didactic problems of the knowledge E / A for the acquisition of competences in SPCT among the learners in Benin. For several years, the implementation of pedagogical prescriptions seems to give rise to difficulties of pedagogical and didactic order and limits the achievement of the objectives of the change of approach. The objective of this research is to analyze the impact of this new approach on the skills of the learners in order to contribute to the improvement of the process of knowledge learning in SPCT. According to our theoretical framework, open and scholastic problems make it possible to know the conception of a learner in relation to an object of study and to study his reasoning. The reasoning of a learner is based on theorems and concepts-in-act. In this article, we study the design of learners in relation to an object of study and their reasoning in solving a problem involving this learning object. To do this we formulate the first research question hereafter: How do the thinking and thinking of Beninese learners evolve in solving problems in the physical sciences? Specifically, we analyze how the transfer of scientific knowledge and the construction and mobilization of empirical knowledge in open problem solving are carried out. In reference to our problematic, we postulate that in the problem solving of physical sciences, the concepts, laws, rules and formulas mobilized by the learners appear respectively as concepts-in-act and the-theorems-in-act. Several absolute and erroneous conceptions determine their distance from the knowledge referred to in the statements and influence the relevance and coherence of the reasoning when the context changes.

### II. RESEARCH METHODOLOGY

### 2-1. The target and the actors

Students in the first year of physics-chemistry (PC), mathematics-physics (MP) or preparatory years of the IMSP have a scientific BAC and represent a valid target for this research in the school institution because this Brings together those who, among the pupils trained, have the minimum knowledge of the knowledge taught in the C and D classes and who wish to pursue their studies in these fields of scientific research. This target represents a selection through the scientific BAC of the students, those for whom the scientific abilities are present. This makes it possible to avoid in the population "intruders" (who found themselves in these classes for social reasons). This target represents a national sampling because it brings together for the most part middle pupils from all the departments and almost the majority of the lycées and colleges of general education of Benin which have the classes of terminals C and D. It makes it possible to gather Learners from the majority of physical science teaching practices in all departments.

### 2.2. Data collection techniques

Three research techniques have been mobilized: investigation, evaluation and treatment.

### 2.2-1. Investigation

This is a fact sheet that analyzes the student's educational background. It gives general information (the BAC series, the composition center and the student's status) and the pedagogical indications (the course material used in SPCT, the name of the teacher and the approach used since second class). This technique made it possible to have data on the representativeness of the student in the population.

### 2.2-2. Data processing

At the outset, our population is open to all students enrolled in the first year of university studies in Scientific Sciences at the University of Abomey Calavi (UAC). For this academic year (2014-2015), the number of enrolled students was about 463 students (235 in PC1, 180 in MP1 and the remainder in preparatory classes for the IMSP.) On return we obtained 372 productions to the number of students present in the classes at the time of the evaluation. To draw the students to take into account in our research, we chose an exclusion policy centered on criteria of representativeness and elimination. Representativeness are related to the survey part and are based on the absence of information, the level of data fill, and the inconsistency in the information given. The criteria for elimination are related to the second part of the survey part on the pedagogical indications and relate to: The condition to be formed in SPCT according to the approach by competence from the class of second or less and in Terminal Scientific class by a certified teacher of more than five years of experience in these classes. This conduction resulted in a reduction in the workforce to 83 written productions. The condition to be formed with a course support built by authors (Inspectors and advisers and teachers) and authorized by the educational system. This condition has reduced the number of 50 written productions. After treatment, the size of the sample studied was thus 50 written productions from the students who took the school curriculum in teaching and learning science with experienced teachers and course materials designed according to the APC and authorized by the institution in charge education.

### 2.3. Evaluation contents and their a priori analysis

The two problems solved by the learners were constructed on the basis of the models of physics problem solving activities defined by J. M Boilevin and Dumas-Carré, (2001). Problem 1 is academic (PS) and involves the object "acceleration" and the ability of the student to identify the fastest of two motorcyclists in a race after identifying the object "acceleration" and compared his Values calculated from the statement data for each of the motorcyclists. It involves the scientific conception of the object and allows the mobilization of a scientific knowledge. Problem 2 is open (PO) and involves "speed" in the "fastest mobile" concept and the ability of the student to analyze the possibility of catching up with a motorcyclist in the pursuit of a Car carrying fraudulent goods. This possibility of catching up is analyzed from the concept of speed. The notion of speed is appreciated by the terms "roll" or "force", "as fast", "faster". This problem brings into play the empirical conception of the object and makes it possible to study, the evolution of the conception, the coherence the transversality of the knowledge.

Tat	ole 1: Defining the c	haracteristic c	ompor	ents of	each ol	bject	ın eac	h cor	ıtext	
		-					_			

	C	1 3	
Objets	concepts	Relation or formula	Target Knowledge
Acceleration in the PS	$\begin{array}{cccc} \text{In} & \text{Km/h} & \rightarrow & \text{Speed} & V \\ \text{In} & \text{h} & \rightarrow & \text{time} & t \\ \text{In} & \text{Km/h2} & \rightarrow \text{acceleration} \end{array}$	→ Accélération     → Accélération     → Accélération	Acceleration of a mobile is the variation of speed with respect to time
Speed in the PO	Rolling constantly Continuously sinking Keep constantlyIncrease speed	Rolling constantly = maintaining speed Continuously stick = increase speed	The speed of a car or a motorcycle depends on how itrolls or runs

**Table N** ° **2:** Correspondences between the concepts of the three levels in the two contexts

Concepts of the 1st level of		Concepts of the 3rd level correspondence
(Scientific context)	life (context of	(Concepts of the two contexts)
Body moving in the terrestrial		The motorcycle, the moving car can be
reference frame (mobile).	the road.	considered mobile.
Constant speed (gain the same	Roll constantly (maintain speed).	If the motorcycle is constantly running then its
distances during the same periods).		speed is constant.
Constant acceleration (gaining the	Roll constantly (maintain speed).	If the motorcycle is constantly running, its
same speeds during the same		acceleration is constant.
periods).		
Superior constant speed.	Roll constantly faster (keep the	If the motorcycle is constantly running faster
	fastest).	then it has a higher constant speed.
Superior constant acceleration.	Frost constantly faster (increase	If the motorcycle is constantly running faster
	the speed more).	thenit has a higher constant speed.

### 2.4. Method of analysis of evaluation data

The production analysis methodology consists of two steps: The first is a categorization of learners by profiles after correction of their productions from an elaborate scale. It made it possible to construct not only the profiles but also to make a statistic of the proportions of learners that are classified in each of these profiles. The second step is a lexical analysis of the learners' productions of each profile. It made it possible to characterize each profile by the reasoning of the learners and their conceptions in relation to the objects of investigation (Moussiliou, 2015). It consists in analyzing word for word the reasoning of their choice. We can therefore characterize by profile the concepts-in-act and the scientific or arbitrary nature of the justification, its relevance and its coherence by articulating the indicators that will be defined. We consider in the matrix of appreciation that reasoning can be qualified in the case of our analysis of:

- → scientific, when it places the object of investigation in the intended design and mobilizes the concepts related to this object;
- → relevant, when it is based on theorems and rules that approach the intended design;
- → coherent, when it mobilizes the knowledge sought under the defined conditions and makes it possible to deduce the intended design;
- → arbitrary when it is based on contradictions and relies on erroneous concepts.

The third step is a sequential analysis of the different stages of reasoning. The sequential analysis (Closet, 1983) consists in exploring concepts and theorem in the different stages of construction to describe a learner's mode of reasoning. It has enabled us to describe in the profiles the modes of reasoning that explain the conceptions of the learners in relation to the objects of investigation and to construct a typology of reasoning of the learners of each profile. In this article we recall the results of the first two steps and we present the results of the third step.

### III. RESULTS OF THE ANALYZE

### 3.1. Result of the lexical analysis: categorization of the profiles and their characterization

In order to categorize learners' productions in problem solving, we proceeded in a first step to a correction of these productions from a scale in the resolution of the PS. We then proceed to a categorization of the productions by assigning the note 1 for a correct answer, 2 for a false answer. A "justification" is given after the letter "j" and "f" for a false justification. The profiles are therefore baptized P(x, a) with x the note designating the nature of the response and x that of the justification. In a second step, we continued in the same way the correction of the productions in PO of each of the defined profiles. This second correction made it possible to define profiles P(x, y, a, v) with the score attributed to the nature of the responses in the OP and x that of the justification with this new categorization we made a statistic of the learners classified in Each profiles and draw up a picture. From the results of this correction, we noted:

Of the learners of the P (1, j) profile who obtained correct answers with a good reasoning we distinguish:-those who still obtain fair answers, that is to say who find that catching-up is possible under conditions C2 and C4 and which justify it by good reasoning. They are classified in the profile P (1, 1, j, j). They represent 8% of the total workforce, ie 4/50,-those who obtain several false answers according to the conditions and who justify it by a false reasoning; They are classified in the second profile for this study and represent 8/50 or 16% of the total workforce. This profile is defined by P (1,2, j, f). Learners of the P (2, f) profile (false and badly reasoned with erroneous conceptions) obtain several false answers in the resolution of the problem, according to the conditions specified in the statement. They constitute the third profile for this study and represent 38/50 or 76% of the total workforce. This profile is defined by P (2, 2, f, f). The following table summarizes this categorization of students from problem solving. All these results of the statistical and lexical analyze of the productions of the learners in the resolution of the two types of problems is summarized in the table below.

**Table N 3:** summarizing the results of lexical analysis of learners productions

Profile	Results	Reasoning	Design of Objects	
		Maintain a scientific, relevant and	Relative Scientific Concepts in (PS)	
		coherent reasoning in both types	(speed / time).Relative Relative	
P(1, 1, j, j)	Righteous in both types	problems.Establish a correspondence	Concepts in (PO) (speed / time)	
(4/50 ou 8%)	problems.	between concepts.	(speed/time).	
		Loses relevance and consistency when	Relative Scientific Concepts in (PS)	
	Righteous in the (PS) and	changing registry.Sometimes there are	(speed / time). Several Absolute	
P (1, 2, j, f) several false results in the		bad correspondences between	Empirical Concepts in (PO) (speed	
(8/50 soit 16%) (PO).		concepts in the construction of	or time).	
		theorems.		
	False in the (PS) and several	Keep a reasoning very unscientific,	Several Scientific Concepts Absolute	
	false results in the (PO)	less pertinent and very incoherent.	in (PS) (speed or time).	
P (2, 2, f, f)		Lack of correspondence between	Several Absolute Empirical Concepts	
(38/50 ou 76%)		concepts in the construction of	in (PO) (speed or time).	
		theorems.	Several Erroneous Empirical	
		Their false results in the (PO).	Concepts in (PO).	

### 3.2. Results of the sequential analysis: learners' modes of reasoning

### 3.2-1. Description of the reasoning modes of learners of the P-profile (1, 1, j, j)

Recall that this profile P (1, 1, j, j) denotes the group of learners who have obtained correct answers with scientific reasoning, relevant and consistent in solving the two types of problems. The lexical analysis revealed relative scientific conceptions defined through the "variation of velocity" and the "variation of time" for the object acceleration in the resolution of the (PS) and relative empirical conceptions having relation to the concepts defining the "Speed" in solving the open problem. They represent 8% of the total workforce. We present below some examples of typical productions of two learners in this profile.

	Réponse	Justification
Pour trouver le cycliste le plus rapidle,	man chi	la moto et la voiture ont donc la même arteme car
calculors les accelerations aprêt an des deux	non j	maniere. or la voiture
ain = Va-Vin som le togodais Vix=Vix=Sokm/h	vitere igales	Vin = Vex "napidité" vitent
t for the second	onij	La moto soule plus vite que la voiture clerc
1 a a - 0 m/o <sup>2</sup>	"rapidite"	Vix > Vix, done elle peut
agn = Vex-Vin also or the 4 km/h et Vex=40 km/h	ntesse supérien	"route est longue
an = to 36103		la mote et la voitire fonce de la nière manière, donc
V2n-V2x = 36 km/h et tou V2n-V4x = \frac{36 10^3}{3600} = 10 m/s	nonj	an = an or la voitue
ATTOMA NOMA 1 m laz	rapidite!	done ti elle court, la voitire
92n = 45045 10m/s = 1m/s <sup>2</sup>		part pas la rettroper
alen le cycliste Ci qui est le venimons	ouij	la voture done aix >done
an= 10  an alers le cycliste Cz qui est le beninois est le plus agride	apriolité ?	l'avance de la voiture car elle accelère plus
let le puis cogne	eccleration"	Que la volume

Extracted from the production (PT E1-1 PS) and the production (PT E1-1 PO) of the learner E21-1 chosen from the type P(1, 1, j, j).

### 3.2-1-1. Rules and theorems-in-act in solving the school problem

**Table 4**: Rules and theorems-in-act in the reasoning of learners of the profile P(1,1,j,j) in the (PS) (Correspondence of the first level)

(						
Theorems-in-Act	Related rules					
$T^2_{1-4}$ If the two wheels run at constant speeds, the fastest is the	If $v1 > v2 \implies$ the C1 is the fastest.					
one with the highest constant speed.	If $v1=v2 \Rightarrow$ both are fast.					
$T_{1.5}^2$ If the two wheels run with constant accelerations, the	If its speed is less than or equal to that of the car, the motorcycle					
fastest is the one with the highest constant value.	will never catch up with the car.					
	If $a1 > a2 \Rightarrow$ the C1 is the fastest.					
	If $a1 = a2 \Rightarrow$ both are fast.					
	If its acceleration is less than or equal to that of the car, the					
	motorcycle will never catch up with the car					

In the resolution of the school problem and for the majority of these learners, a correspondence between the concepts-in-act of the first level of correspondence (scientific context) has brought them closer to the scientific conception of the object "acceleration". This conception was obtained by a relation (mathematical solution) established between the concepts-in-act obtained at the first level of correspondence. The learners' results have been justified by these concepts-in-act associated with each of these objects in the scientific context and remain there exclusively. The object "acceleration" represents for the learners, the variation of the speed with respect to that of the time. The "fastest moving" concept is defined with respect to the object and represents the motive whose acceleration has the greatest value. The table below presents some theorems that are deduced from the justifications of the learners of this profile associated with them.

### 3.2-1-2. Rules and theorems-in-act in the solution of the open problem

In the resolution of open problems, for the majority of these learners, a correspondence between the concepts of the two contexts has brought us closer to each of the empirical conceptions of each object. The results of the study were justified by the concepts associated with each other in the context of life. Learners consider the terms "constantly rolling" and "constantly running" as elements of justification. They therefore define the speed as the variation of the speed with respect to time and mobilize it to analyze the possibility of

catching up. This is the first step in the realization of the concept of a constantly increasing faster or more rapidly increasing speed. The table below presents some theorems that are deduced from the justifications of the learners of this profile associated with them.

**Table 5:** Rules and theorems in the reasoning of learners of the P(1, 1, j, j) profile in the (PO) (correspondence of the second level)

Theorems-in-Act	Related rules
- T22-15 If the motorcycle and the car drive constantly then	If the motorcycle is constantly running faster then it can catch
they keep their speed, so the fastest one will be the one with the	up.
fastest speed.	- If the motorcycle constantly runs less or as fast then it will not
- T22-16 If the bike and the car are constantly dying then they	catch up.
are constantly increasing their speed so the fastest is the one	- If the bike is constantly running faster then it can catch up.
whose increase in speed has the highest value	- If the motorcycle is constantly running less or as fast then it
	will not catch up.

For the small minority of these learners, a correspondence between the respective concepts-in-act of the two levels of correspondence (scientific context and context of life) brought closer to the conceptions aimed for each object in the utterance. This conception was obtained by a relation (complete solution) established between the concepts of the two contexts. The learners' results were justified by these concepts-in-act and the-in-act theorems associated with each of the objects. Thus, these learners consider the car and the motorcycle to be mobile, to drive constantly faster or to constantly rush as elements of justification. They therefore consider the rapidity (variation of the speed with respect to time) as the acceleration of each mobile, but do not specify or designate it by another term, and use them as an index of the accuracy of the catch-up condition. This "acceleration" speed was obtained by a relation (complete solution); "The speed of the motorcycle or the car represents the acceleration of each of these motives" established between the concepts-in-act obtained respectively in the two levels of correspondence "roll constantly faster or have the greatest speed" with " Have a higher constant velocity"; Constantly faster or increase its speed "with" having a higher constant acceleration ". To justify their results, they mobilize the theorems-in-act of the two levels and remain in the scientific context to use the object "acceleration" and / or in the context of life to translate it by "rapidity" or other terms Similar. The table below presents some theorems that are deduced from the justifications of the learners of this profile associated with them.

**Table 6:** Rules and theorems in the reasoning of learners of the P (1, 1, j, j) profile in the (PO) (correspondence of the third level)

- T22-17 If the motorcycle and the car drive constantly so fast, - If	- If the constant speed of the motorcycle is higher than that of
they keep their speed and therefore have constant speeds. The the	
fastest will be the one whose speed or constant speed is the greatest.  - T22-18 If the bike and the car are constantly dying then they constantly increase their speed and therefore each have a constant acceleration. The fastest is the one whose increasing  -If	If the constant speed of the motorcycle is less than or equal to that of the car then it will not be able to catch up.  If the constant acceleration of the motorcycle is greater than that of the car then it will catch up  If the constant acceleration of the motorcycle is less than or equal to that of the car then it will not be able to catch up with

### 3.2-1-3. Description of the reasoning modes of learners of the P (1, 1, j, j) profile in the PS resolution

In solving the school problem, all learners calculate constant values of the object of investigation and compare them. We can therefore conclude that all the correct results are obtained by comparing established mathematical solutions, which represent the scientific conceptions of these learners with respect to objects. Thus, for the majority of these learners, the object "acceleration" represents the variation of the speed of the mobile relative to that of the time, so the concept "the fastest mobile" is defined by comparison of the constant value of this object Calculated from the data of the statement. The results of the learners in solving this problem were justified by a reasoning centered on the comparison of the values calculated for the object acceleration (Model (CVC): Comparison of the calculated values). In this model, the reasoning of the learners is based on their calculation; the learners mobilize data from the utterances to establish a relation (concept-in-act). The result is justified by comparing the values calculated for each object. In this model, we distinguish two operational steps: a first stage in which the learners identify and calculate the object of investigation and a second step which is that of the comparison of the calculated constant values and the decision-making. This model is mobilized by these learners in solving the school problem and is well illustrated by the extract of the production (PT E21-1 PS) of the learner E21-1 in the lexical analysis.

3.2-1-4. Description of the reasoning modes of learners of the P (1, 1, j, j) profile in the OP In the resolution of the open problem, the reasoning was the same for most of these learners. In their reasoning

formulate the solution by matching the terms associated with the object in the scientific context of each object and those associated with it in the daily life of the learners. So for these learners, the object "acceleration" corresponds to the "speed". This speed is seen in terms of constant speed or constant acceleration according to the parameters characterizing the moment. Thus the scientific concept "the fastest mobile" corresponds in the context of the problem open to the "condition of catching up by the motorcycle" and is defined by concepts defining the speed of the motorcycle or the car according to the race condition. The correct results of the learners in the resolution of the open problem were justified by a reasoning centered on a model of correspondence of the terms and concepts defining the speed of each machine (Model (CTC): Correspondence between theorems and concepts). Learners make a correspondence between their concepts and theorems-in-act of the two registers to define the relevant object for the reasoning in each concept. In this model, learners establish correspondences, each of which leads to solutions. Here we distinguish three levels of correspondence: a first level of correspondence between the kinematic concepts and those of the context of life and the construction of concepts-in-act; A second level of correspondence between the concepts-in-act of the two contexts and construction of the-the-in-act to formulate the solutions; A third level of correspondence between theorems-in-act to formulate the complete solution. This mode of reasoning is mobilized by these learners in the resolution of the open problem and is well illustrated by the extract of the production (PT E21-1 PO) of the learner E21-1 in the lexical analysis.

### 3.2-1-5. Typology of reasoning of learners of the profile p (1, 1, j, j)

The following table shows the different abilities of the learners in this profile that characterize the different models.

Models		School Problem (PS)	Open Issue (PO)
	1 <sup>ere</sup>	Identification and calculation of the	The statement does not allow this model.
CVC: Comparison of	étape	acceleration values a1 and a2 of the two	No calculation of values.
calculated constant	-	cyclists.	
values.	2nd	Comparison of the values of the calculated	No comparison or solution.
	step	accelerations and choice of the fastest	
		cyclist.	
CTC:	1ere	The two cyclists racing on the track can be	The motorcycle, the car moving on the road can
Correspondence of	step	considered mobile in motion in the	be considered as moving mobiles in the
theorems and		terrestrial reference frame.	terrestrial referential.
	2nd level	Acceleration of a mobile is the variation of	If the motorcycle is constantly running then it
		the speed with respect to time $x = \Delta V / \Delta t$ .	has a speed
			If the motorcycle is constantly running, it has an
			acceleration.
	3rd level	If a cyclist has the highest value of	If the motorcycle is constantly moving faster
		acceleration, then it is the fastest.	then it constantly has a higher speed.
			If the bike is constantly running faster then it has
			a higher acceleration.

**Table 7:** Describing the different modes of reasoning for learners of the P-profile (1, 1, j, j)

**3.2-2.** Description of the reasoning modes of learners of the P-profile (1, 2, j, f) Recall that this profile P (1, 2, j, f) denotes the group of learners who have obtained correct answers by a pertinent and coherent scientific reasoning in the resolution of the (PS) and several intermediate results just and false in the resolution of (PO). Lexical analysis revealed relative scientific conceptions defined through the concepts of "variation of velocity" and "variation of time" for the object acceleration in the resolution of the (PS) and several empirical absolute or erroneous conceptions related to the One of the concepts defining "speed" in solving the open problem. They account for 16% of the total workforce. In the resolution of the school problem and for the majority of these learners, such as those in the P (1, 1, j, j) profile, a correspondence between the concepts-in-act of the first level of correspondence Allowed to approach the scientific conception of the object. The rules and theorems defining their mode of reasoning for these learners are the same (T21-4, T21-5) as those of the P (1, 1, j, j) profile in the (PS). We present below some examples of typical productions of two learners in this profile.

Réponse	Justification	Réponse	Justification
Out No	Varione la deux conument voulent à la mene rapidite voience	Non, elle ne. paut pas ratholiper	La viterre de da moto est egale à celle de la voiturg or la voiture à démarrer après la moto rapideté " "vinteratile"
Oui E	lantone de la moto est plu grande pre celle de la rapidite arterne!	Dui elle jout rattraper	da viterse de la moto olepasse celle de la voiture a malgré que la voiture à demarrer avant lui.
Hon j	Restriction Negridate 11	Non, elle ne peut pas reuten fer	La viterse de la moto Les deux fonces de constamment, "fincer contament & "rapidite!"
ani	france de la moto explus france for coste de la	our, elle sent	
3	Restrotion" "napidite" "oriene"	-j	" foran plus que "

Excerpts from the learner E<sub>2-2</sub> and (PT E2-2 PO) of the learner E<sub>2-2</sub> chosen from the type P (1, 2, j, f)

### 3.2-2-1. Rules and theorems-in-act in the solution of the open problem

In the resolution of the open problem, the majority of these learners match the Concepts-in-Act of the second level of correspondence (context of life) to define several empirical conceptions of the object. In their justification they formulate solutions by establishing a correspondence relationship between their concepts-inact at the second level of correspondence and base their reasoning on the relevance of the concepts "roll" and "go" for some and "as fast" and "Faster" for others. The learners' results were justified by concepts associated exclusively with the object in the context of life. Some learners also consider the terms "constantly rolling, steady running" as elements of justification, but they define timeliness by the nuance of the terms "rush" and "roll" and mobilize the  $T^2_{2-19}$  theorems and  $T^2_{2-20}$  for which the fastest mobile is the one that rolls or which moves the most. This justification refers to a restriction of the notion of speed to speed. This is at the origin of the false intermediate results obtained by the learner E22.1 whose production shows a relation between their concepts-inact obtained at the second level of correspondence Rolling constantly or gaining more speed, running steadily or gaining more speed). Other learners also consider the terms "as fast" and "faster" as elements of justification, but they define the rapidity with the absence of nuance on the terms "also" and "plus" and mobilize theorems-In-act  $T^2_{2-21}$  and  $T^2_{2-22}$  for which the fastest moving is the one that rolls or which fastens fast. This justification suggests a confusion of "speed" speed with an advance or a delay in the start or a relevance of the terms and is at the origin of the false intermediate results obtained by the learner E22-2 for the second condition of "A share and the last two terms of other shares. This production shows a poor correspondence between the concepts-inact of the second level of correspondence (Ride fast or gain more speed, Rush constantly or earn more). The table below presents some theorems that are deduced from the justifications of the learners of this profile associated with them.

**Table 8:** Rules and theorems-in-act in the reasoning of the learners of the profile P(1, 2, j, f) in the (PO)

Theorem-in-acts	Related rules		
From two machines in the race, the fastest	If the motocycle		
- T <sup>2</sup> <sub>2-19</sub> one whose speed or speed is the greatest or which is	- rolls constantly, so its speed is higher than that of the		
constantly rolling the most $\mathbf{E}^{2}_{2-1}$	car.		
- T <sup>2</sup> <sub>2-20</sub> one whose speed or speed is the fastest or fastest.	- rolls fast, then its speed is higher than that of the car.		
$E_{2-1}^2$	- runs constantly, so its speed is higher than that of the		
- T <sup>2</sup> <sub>2-21</sub> one whose speed or speed is the greatest or which is	car.		
the most powerful <b>E</b> <sup>2</sup> <sub>2-2</sub>	- goes fast, then its speed is higher than that of the car		
- T <sup>2</sup> <sub>2-22</sub> the one whose speed or speed is the biggest or which	Then she can catch up.		
fastens fast. E <sup>2</sup> <sub>2-2</sub>	-		

### 3.2-2-2. Description of the reasoning mode of learners of the P-profile (1, 2, j, f) in the OP resolution

In the solution of the open problem, reasoning leads to the same model for most of these learners. To justify their results, learners base their reasoning on the concepts of life related to each of the objects evoked by a correspondence between the scientific context of each object and the real concept in the daily life of the

learners. So for this minority, the object of investigation corresponds to the real term associated with the daily life of the learner which is "rapidity". The notion of speed is seen in terms of speed for the most part and is appreciated through several concepts evoked in the problem. Thus the scientific concept "the fastest mobile" corresponds in the context of the problem open to the "condition of catching up by the motorcycle"; Only the senses given to the concepts which define it in the correspondences of the second level allow its restriction or its deformation according to the race condition. The false intermediate results of learners in solving the open problem have been justified by reasoning centered on a model of correspondence of terms and concepts that reduce speed to speed or confuse it with other objects. In their justifications, the learners of this profile mobilize the reasoning of Model (CTC).

**3.2-3.** Description of the reasoning modes of learners of the P-profile (2, 2, f, f) Recall that this profile P (2, 2, f, f) refers to the group of learners who obtained false answers with a very unscientific, less relevant and totally incoherent reasoning in the resolution of (PS) and intermediate results just and false In the resolution of the (OP). The lexical analysis revealed several absolute scientific conceptions defined by the concepts of "variation of velocity" or "time" for the object acceleration in the resolution of the (PS) and several empirical absolute or erroneous conceptions relating to one Or the other concepts defining "speed" in solving the open problem. They represent 76% of the total workforce. We present below some examples of typical productions of two learners in this profile.

- 1 12 12 Officely	Réponse	Justification
V1= 50 6m/h = 13,88m/5	oui	Car la moto roule vite que
The state of the s	0	la voiture et de plus la
V2 = 48m/A = 121/m/5	5	Voiture est chargée des marchan-
10 44 44m/6		sises. Cela l'empêche so vile
V3 = 40 &m/2 = 11,11m/5	(0)	Car la moto route plus vite
- 10	oui f	que la voitine. et de plus les
91 = 13,188	0	Pranduleusemen de marchandes
10		qu'elle transporte l'engrèche de Vite roules. Por de
70		
= 1,388	ow &	Bien haire que la moto fonce auxi Ville que la Voitare. Mais
	9	comme la voulles plane, devil
a= 1,388 m/s2 az = 1 m/s2		stassez longue et assence de bofon - c'ele la ragnathape.
= 10lle du beninots,		route
Cocceliration du togolais est supérieure à celle du beninois,	(Oni)	Lar la route est plane, droite et anez longue. Se plus, l'absorce
occernation of sur rapide	Om ?	et anez longue. Se plus, L'absorce
on peut walking dire que l'est le tegolair qui est plus rapide	0	de bafar prouve qui elle
or from the second	The second second second	nattrape : conte
		The state of the s

Production extracts (PT E3-1 PS) E23-1 and production (PT E3-1 PO) E23-1 chosen from type P (2,2, f, f)

T. do CAL	Réponse	Justification
Le togolais C1 a une vitere constante de 5064	non	car la moto ne pas rouler cons-
mal of the displace of the colores	8	la voiture
une ortene mascimale de 40 km/h	non	car la moto ne pert pas nouler plus trite que la
Jac Jan doux Inches bout company	6	nature des engin 14
I togetain en eleptus pagnet	non oui	La moto est Bnw La voiture est 505 on OPEL donc elle peut rattraper
time ut avance		La moto peut foncer plus que la voiture
Car pa or tem or mperieur a 40km/h	Qui	car elle n'a pas de marchandise

Production extract (PT E3-3 PS) E23-3 and production (PT E3-3 PO) E23-3 selected from type P (2,2, f, f)

### 3.2-3-1. Rules and theorems of learners of the P-profile (2, 2, f, f) In solving the school problem

In the resolution of the school problem, some learners define the two objects by considering one or the other of the two concepts related to each of them. In the case of solving the problem of schooling involving the object "acceleration", in order to choose the fastest cyclist, some learners identify the data of the statement as an object of investigation and mobilize them in the reasoning. In this case, learners' reasoning is also based on a comparison of the data identified for each cyclist (the speed of each mobile or the duration of the journey). They therefore identify the fastest mobile by the result of this comparison. This is the case of the learner E23-3 whose production (PT E3-3 PS) shows a justification centered on the comparison of the speeds of each of the cyclists. Thus the false results obtained can be translated by the use of non-operative theorems-in-act under specific conditions of the utterance. The model of reasoning of the learners of this profile in the resolution of the school problem is the CVD model: Comparison of the given values. The table below presents some theorems that are deduced from the justifications of the learners of this profile associated with them.

**Table N**  $^{\circ}$  **9**: Rules and Theorems-in-act in reasoning of the learners of the profile P (2, 2, f, f) in the (PS)

Theorem-in-acts	Related rules
$T_{1-6}^2$ Of two movers, the one whose speed has the constant value or	If $v_1 > v_2 \Rightarrow$ the C1 is the fastest
whose travel time is the greatest accelerates the most.	If $t_1 > t_2 \Rightarrow$ the C2 is the fastest
	If $v_1 = v_2 \Longrightarrow both$ are fast
$T^{2}_{1-7}$ If two mobiles or gears have the same constant speed or the	If $t_1 = t_2 \Longrightarrow both$ are fast
same travel time, none of them is the fastest. If $v1 > v2 \implies$ the C1 is	If its speed is less than or equal to that of the car, the
the fastest	motorcycle will never catch up with the car

In solving the school problem, other learners in this profile define these objects by considering parameters related to each of them. Learners mobilize data from the statement to establish a relationship (concept-in-act). They justify their answers by comparing the constant values calculated for each object. They make a comparison of the constant values calculated from the identical data (the speed of each mobile or the duration of the journey). They consider this constant which is often the object "acceleration" or "speed" but often poorly calculated, as the only element of comparison and choose the fastest moving from the result of this comparison. The value of the acceleration is obtained by the ratio of the velocity over time or by the ratio of the variation of the velocity with respect to that of the time. The false results are due to conversion errors, calculation errors, poorly understood data and especially poorly formulated formulas. The model of reasoning of the learners of this profile in the resolution of the school problem is the model (CVC): Comparison of the calculated values. Only that these learners limit themselves to the first step of a good identification of the object of investigation and the errors of calculation, conversion and choice of non-operational formulas prevent the second step. This is the case, for example, of the learner E23-1 whose production (PT E3-1 PS) shows a justification centered on the comparison of the values of the acceleration calculated for the Togolese cyclist whose speed is constant. These learners mobilize the CVC model with theorem-in-act that are done on erroneous conceptions. The rules and theorems in-act are defined in the table below.

**Table N**  $^{\circ}$  **10:** Rules and theorems-in-act defining the mode of reasoning of the learners of the profile P (2, 2, f, f) in the resolution of the (PS)

Theorem-in-act	Related rules
- <b>T</b> <sup>2</sup> <sub>1-8</sub> The fastest accelerator is the fastest.	$-a_{1x} > a_{2x}$ then the cyclist $C_1$ is the fastest.
- $T^2_{1-9}$ The duration of the journey being the same then, the	- If $\Delta t_1 = \Delta t_2 \Longrightarrow$
fastest mobile is the one whose maximum speed is the largest.	$a_{1x} > a_{2x} \text{ si } V_{1x} > V_{2x}$
	$a_{1x} < a_{2x} \text{ si } V_{1x} < V_{2x}$

# 3.2-3-2. Rules and theorems-in-act of the learners of the P-profile (2, 2, f, f) in the solution of the open problem

In the resolution of the open problem, the learners of this profile define the two objects by considering the meaning given to certain concepts in their daily life. The relationship to the students' knowledge is based on the meaning and knowledge they have of a few concepts already mobilized in everyday life. This sense removes them from the objects and is considered pertinent in the reasoning of their choices in the resolution of the open problem. For example: for the theme of the study of the movement of a mobile, the concept "the fastest mobile" is defined through the meaning given to the following words and expressions: rolling and rushing; Faster than, as fast as, less quickly than, or bearing on the brand of used gear. This is the case, for example, of the learner E23-3 whose production (PT E3-3 P0) shows a justification centered on the natures of the machines (BMW, PEUGEOT) for the first two conditions, on their marks for the third Condition and their weights in the last condition. This is also the case, for example, of the learner E23-1 whose production (PT E3-1 P0) shows a

justification centered on the weights of the gears for the first two conditions, on the state of the roads for the two conditions Last condition. Thus another model of reasoning of the learners of this profile in the resolution of the open problem is the model (PSCQ): Relevance of the meaning given to the daily concepts of the learners. The false results of these learners in this problem were obtained by the following theorems-in-act:

- $-\mathbf{T}^{2}_{2-23}$ The fastest moving vehicle is the fastest one;
- $-\mathbf{T}^{2}_{2-24}$ The fastest mover is the one that moves faster;
- $-\mathbf{T}^{2}_{2-25}$ The speed of a craft depends on its brand.

### 3.2-3-3. Different models of reasoning of learners of the P-profile (2, 2, f, f)

In their justifications, the learners of this profile usually mobilize three different modes of reasoning, the characteristics of which are presented in the table below.

**Table N**  $^{\circ}$  **11**: describing the characteristics of the learners' different modes of reasoning Of the profile P (2, 2, f, f)

Model		Situation N°2	
		Problem 1	Problem 2
CVD: Comparison	of given constant	Comparison of speeds over journey	No solution or hazardous solution without
values		times	justification
CVC:	First	Incorrect identification and	The statement does not allow this model.
Comparison of	Step	calculation of the acceleration	No calculation of values.
calculated		values a1 and a2 of the two cyclists.	
constant values.	Second	Comparison of erroneous values of	No comparaison or solution.
First	Step	calculated accelerations and poor	
		choice of the fastest cyclist.	
PSC: Relevance of the meaning given		Rationale based on the nationality	Rationale based on the meaning of the
to certain concepts in the everyday life		of cyclists (Benin or Togo)	concepts: Driving, driving faster, slower or
of pupils			on the brand of vehicles (BMW, Peugeot) or
			the nature of the gear (car or motorbike)

### 3.3 Synthesis of analyzes

The relative scientific and empirical conceptions characterizing the P (1,1, j, j) profile and their results are obtained by a comparison of the constant values calculated in the PS and preceded by a correspondence between the theorems and concepts associated with each Of objects in both contexts. The mode of reasoning is the same in both situations and leads to the same result. The absolute or erroneous empirical scientific conceptions characterizing the P (1, 2, j, f) and P (2, 2, f, f) profiles and their false results are obtained by different modes of reasoning according to the nature of the problem in each Situations. In SPs and POs that use calculations, false results come from a comparison of the incorrectly calculated or calculated values from the erroneous concepts, or that of the constant values given in the utterance. In PS and PO that do not use calculations, false results come from restriction, distortion, or confusion in the relevance of the meaning given to the concepts that define the object of investigation. All these parameters depend on the ambiguity of the terms defining the concept and the complexity of the associated knowledge. It seems to us that good conceptions in relation to objects of investigation favor coherent reasoning and that absolute and erroneous conceptions favor very unreasonable and arbitrary reasoning.

### IV. DISCUSSION

To summarize the comparative analysis of the results of the two studies, we can conclude that the Beninese learners holding scientific BAC and regularly enrolled in the first years of university studies in the scientific fields do not have the same scientific aptitudes. In the solving of problems involving knowledge of their levels of study, the analysis of their different productions reveals three different profiles, each characterized by their conceptions and their modes of reasoning. This analysis clearly shows their concepts-inact to build knowledge, the rules and formulas on which their different concepts-in-act are founded are at the origin of several conceptions and several modes of reasoning in each of the profiles. From this analysis we can conclude a close relationship between the reasoning of the learners and their conceptions. Definitions, formulas and laws mobilized by learners in the resolution of both types of problems appear here as properties or relationships that are retained when the conditions or contexts of investigation change. To a certain extent they represent operative invariants (Vergnaud, G. 1990). Because certain concepts defined by the learners are held only relevant or not to develop reasoning are not susceptible to truth or falsity. They appear as concepts-in-act. For the majority of learners, rules and formulas mobilized in the construction of reasoning favor theorems or propositions held to be true in problem-solving activity. Because their constructions are animated by conceptsin-act, they are therefore susceptible to truth or falsehood according to the conditions of investigation and appear as theorem-in-act. This analysis confirms our hypothesis and justifies that the close relationship between the conceptions and the mode of reasoning of the learners are based on concepts and theorems in-act. The nature of each of these relationships depends on the complexity of the study objects and the nature of the problem involved. The results of our analysis confirm those of Giordan (1996) on conceptions, of Klein (2003) and Kahn (2006) on the place of reasoning in the development of a competence. This analysis confirms the results of Vygotsky (1978) show that the daily concepts used to bring the concept of speed into the open problem are overloaded with elements (brand, nature, nationality and others) and scientific concepts (distance, Time and others) that define the acceleration object in the scientific problem are too theoretical and have no images in the reality of the learners.

### V. CONCLUSION

In the new teaching / learning approach to concepts in the physical sciences in Benin, the implementation of institutional prescriptions should not only enable the different conceptions of an object to be built among all learners, but also to develop the same methods Of reasoning in the resolution of a problem involving this object. The analysis of the different productions of the learners we have made shows that in the resolution of the problems involving the object "acceleration", the rules and theorems mobilized by most learners are theorems and concepts-in-act Which are due to absolute or erroneous conceptions in relation to objects and which occasion several modes of reasoning in the resolution of the different types of problems involving the same object of knowledge in various contexts. Restrictions, confusions and misalignments between the different solutions are the true sources of the different intermediate results, of the different conceptions observed in the change of context. According to our theoretical references, conceptions and reasoning are necessary to be competent. In the present conditions of its implementation (erroneous conceptions in relation to objects of knowledge favor several reasoning in the solution of a problem), it seems to us that the didactic engineering prescribed and implemented by the teachers for years allows In spite of the willingness of the actors, to train a learner capable of undertaking scientific research in university studies. After several years of training and retraining, if the objectives are not always attained, the question of examining the framework of the didactic tool is necessary. Finally, we think that it would be interesting in our context to envisage later the construction of a more effective tool in order to generate in the learner all the conceptions aimed at an object that will allow him to develop a scientific reasoning And coherent in problem solving.

### REFERENCES BIBLIOGRAPHIQUES

- [1]. Actes du Forum National sur le secteur de l'Education. Rapport. FNE (Octobre, 2007). DIP/DES/MESFTP, Porto-Novo, Bénin.
- [2]. Basile, Agbodjogbe. (1993). L'implémentation des nouveaux programmes par compétences au Bénin : des textes officiels aux pratiques d'enseignement. Analyses didactiques en EPS et en SVT en classe de 5ème. Toulouse II : Thèse de doctorat Université. Pp47-60.
- [3]. Boilevin, J.-M. & Dumas-Carre, A. (2001). Un modèle d'activité de résolution de problèmes de physique en formation initiale d'enseignants. Aster 32, 63-90
- [4]. Closset, J. L. Le raisonnement séquentiel en électrocinétique. Paris. Thèse Université Paris ,1983.
- [5]. Dumas-Carré, A., Goffard, M. & Gil-Perez, D. (1992). Difficultés des élèves liées aux différentes activités cognitives de résolution de problèmes. Aster 14, 53-75.
- [6]. Dumas-Carre, A. & Goffard, M. (1992). Utiliser des problèmes papier/crayon? Oui, mais autrement. Bulletin de la société Française de physique, 87, 17-20.
- [7]. Fabre M. & Vellas E. (dir.) (2006). Situations de formation et problématisation (pp 18-30). Bruxelles : De Boeck.
- [8]. Rey, B., Carette, V., Defrance, A., Et Kahn, S. Les compétences à l'école ? Apprentissage et évaluation. Bruxelles, De Boeck, 2003.
- [9]. Vergnaud, G. (1990). La théorie des champs conceptuels. Recherches en didactique des mathématiques, 10(2-3), p. 133-170
- [10]. Vergnaud, G. (1991). Langage et pensée dans l'apprentissage des mathématiques. Revue française de pédagogie, n° 96, p. 79-86
- [11]. Vergnaud, G. (1994). Apprentissages et didactiques, où en est-on? Paris, Hachette.
- [12]. Viennot, L., & Raison, S. (1992). Students reasoning about the superposition of electric field.
- [13]. Vygotsky, L. S. (1978). Mind in Society. Cambridge: MA: Harvard University Press: