

An Experimental Setup for Teaching Newton's Law of Cooling

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ABSTRACT: *When differential equations are presented in most of Mexican universities mathematics courses, it is common that students have some problems identifying relationship between these equations and real world. In this work an experimental setup for teaching the most important features of Newton's law of cooling is presented. A simple electronic circuit is implemented in order to let the students contrast the solution of the linear differential equation that characterizes the phenomenon with results obtained in circuit measurements. In order to validate the effectiveness of the proposed method, experimental and control groups are set.*

Keywords: *Teaching, learning, cooling dynamics, academic performance, electric circuit*

I. INTRODUCTION

An age of great technological advances has led the modern world, especially in electronics, computer sciences and robotics, disciplines that have been used effectively not only to improve the quality of life worldwide, but also as a teaching methodology in schools [1]-[4]. The advances in science and technology can also be used to transmit efficiently the knowledge to students through appropriate practical and computer-based activities which allow them to corroborate theories exposed in daily life [5].

In Mexican universities, the field of mathematics is frequently handled without considering appropriate practices to prove equations and concepts in the real world, although there are many interesting phenomena where mathematics can be applied. Among differential equations exist several important applications, one of them is related with Newton's law of cooling, which is presented in many common phenomena, for example in the manufacture of an industrial metallic piece and in the cooling of a cake fresh from the oven [6].

Precedents which caused this work have their origin in difficulty presented when teaching differential equations in university programs, because of dryness presented for lack practical implementation with elements that students can detect in a tangible and explicit way. This problem has been reported by several university teachers who have taught the subject of differential equations during several generations. All this generated the idea to use electronic components to create a design that allows a computer to obtain temperature readings and built graphs which can be compared against the theoretical curve.

The general objective was to improve comprehension of students related to Newton's law of cooling by verifying results obtained theoretically and those obtained in real measurements. To accomplish the goal, a didactic prototype using electronic components for verifying temperature variation of a body exposed to an environment with a different temperature was made. It was also important to couple the signal of ADC to PC to storage data and for its subsequent interpretation [7]. It was essential to select the experimental and control groups in order to compare academic performance in both at the end of the test.

The use of a didactic prototype that allows the verification of Newton's law of cooling facilitates students comprehension in this subject of differential equations, so it saves time in data acquisition and graphs generation, showing an automatic verification system between practical and theoretical cooling curves. These developments justify the implementation of physical, electronic and computational elements that students can perform to motivate their own knowledge, generating a propitious environment to raise the level of understanding of this important subject in real world and make the class more pleasant.

Nowadays teaching of mathematics is not stock-still, but it has changed to more dynamic shapes that involve students participation, so they can obtain their knowledge by reasoning which is generated through interesting and even funny activities applied in daily life. Pedagogy should allow to build and acquire knowledge moreover lead mathematical thinking and logical, numerical and space reasoning, like so awake a natural interest while facing challenges for the solution of problems [8].

Learning mathematics at any level implies an attractive challenge at all times, and assuming it ideally should be enjoyable and not be a heavy burden for the student, but rather be taken as an opportunity where the search of solutions pursues no other objective than to give satisfaction to those around it [9].

Metacognitive processes, which invite individual reflection and have a unifying character that underlies a profound conception of the learning processes, cannot be achieved without the optimal conditions to trigger the natural flow of intellectual processes in the individual. In this sense, the efforts of teachers should be aimed at creating the appropriate learning environments to arouse the student's interest in the programmatic contents of

the subjects, and one of the ways to achieve it is through the experimental implementation that corroborates theories exposed in class [10].

The theory-based school interventions are very different from those based on practice, the theorist whose work is summarized in the text only tries to break down the basic laws of memory, which would only allow explaining the mechanical repetition of meaningless concepts. Even in these days, echoes of these situations are found in a great quantity of materials designed to improve the mental abilities; there is also an important number of teachers who resort to their own knowledge and who rely on popularizing methods that will improve mental processes [11]. The inclination that has been adopted in questions of thought, underlies the theory of multiple intelligences, which has been adopted in the last decades [12].

Most math teachers generally show a marked tendency towards the use of annoying and arid algorithms, instead of establishing practical, simple and interesting examples, they present their students real enigmas whose meaning is not understood or deciphered however, the contrast between theoretical mathematics and its relation to practice constitutes a very high step in the understanding of concepts and awakens interest in the majority of students [13].

Many people have the deep-seated idea that mathematics is boring, cold and difficult, although some have a very different view of them as a warm, beautiful and exciting subject, it is all a matter of focus and the fact that mathematical relationships are in the natural world and that men live immersed in them, constitutes an undeniable truth that will be used by the mathematician to offer those who have a negative concept of this important science, a portal that allows to change their perspective by revealing their presence everywhere, thus corroborating its application in practice [14].

This concept in which mathematics is presented as a living, tangible and regulating entity of the relationships that exist between the various elements that constitute the life of man, becomes evident through the 17 equations that changed the world. This set of equations reveal a special brightness that should emerge in the students by meditating on the importance of numerical relationships and their characterization in the mathematical models that explain the reality of the world in which they live [15].

II. METHODOLOGY

During the course of the differential equations, two groups of students were selected, one that received the classes in a traditional way, and another which was treated using the proposed teaching-learning method. Each group was of 20 students, which were examined at the end of the exercise to measure their understanding of Newton's law of cooling.

The students were taken from groups already established, but with very similar general characteristics, since they were students of professional level and of the same degree, in this way, a possible bias generated by its heterogeneity diminished. The first thing that was done, was to explain the problem of Newton's law of cooling in the classroom, then the differential equation was modeled and solved. In addition some exercises were performed. All this was applied for the two work groups.

Afterwards, the students of the experimental group had a practical experience in which they verified the behavior of the cooling system of Newton. Working groups were formed of four students, who implemented a circuit that was constructed and managed with the following materials: a resistance of 47 Ω to 5 W, an experimental board, a switch, a temperature sensor LM35, a multimeter, a power source, a data acquisition board, a PC and electrical wire as it is shown in Fig. 1.

The thermal radiation of the resistance due to the current flow falls on the body of the LM35, which sends a voltage signal proportional to the temperature (10 mV for each degree centigrade), which is transformed by the ADC to a digital format and it is also sent to the computer for its processing.

Once the circuit was built, the students first measured the temperature of the environment and recorded it (T_A), then closed the switch and waited until the resistor temperature was 75°C, once this was accomplished, the switch was opened, so the initial temperature $T(0)$ was 75°C. Five minutes later the new body temperature was measured, then with the initial data $T(0)$, the ambient temperature T_A , and the temperature recorded at five minutes $T(5)$, the students calculated the constants of the exponential equation that solves the differential equation, (1), where "t" represents the time, "T(t)" is the resistor temperature in function of time, "c" and "k" are constants of the system, and " T_A " is the environment temperature [16].

$$T(t) = ce^{kt} + T_A$$

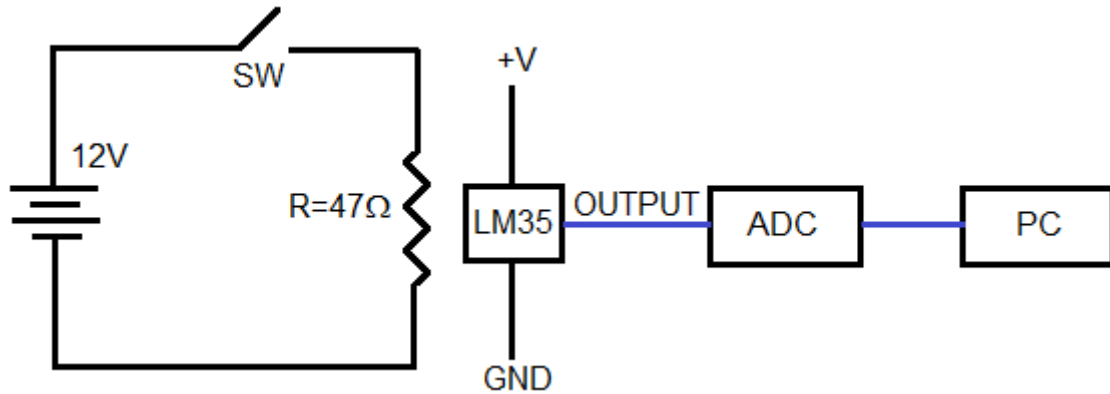


Fig. 1. Experimental setup to prove differential equation related to Newton's law of cooling.

Once the constants were obtained, the students calculated the temperature values that the resistor would have at 10, 15, 20, 25 and 30 minutes. Then the temperatures were recorded every 5 minutes and the results of the measurements were compared with those predicted by the mathematical model. Finally, after a few days an evaluation was applied both to the students of the control group and to the experimentation group, which consisted of three sections, the first one had to do with the understanding of the dynamics of the system, the second referred to problems where it was necessary to calculate temperatures and the last one in relation to problems where the calculation of the times was required.

III. FINDINGS AND COMMENTS

Table 1 presents the temperature values recorded and calculated, as well as the deviations thereof, taking as reference an ambient temperature of 30°C. Fig. 2 shows a comparison between graphics of recorded and calculate temperature.

Table 1. Registered and calculated temperatures in the experiment.

| Time (minutes) | Recorded temperature (°C) | Calculated temperature (°C) | Variation (°C) |
|----------------|---------------------------|-----------------------------|----------------|
| 0 | 75 | 75 | 0 |
| 5 | 48.8 | 48.8 | 0 |
| 10 | 38.4 | 37.85 | 0.55 |
| 15 | 34.6 | 33.28 | 1.32 |
| 20 | 33.1 | 31.37 | 1.73 |
| 25 | 32.5 | 30.57 | 1.93 |
| 30 | 32.2 | 30.23 | 1.97 |

As regards the students' performance of the experimental and control groups in relation to Newton's cooling problem, Table 2 presents the results, where the understanding column of the dynamics of the system refers to the first section of the test applied to both groups, whose questions were directed to the understanding of the mathematics of variation (Dolores et al., 2007) and the behavior of the phenomenon itself. The temperature calculation column shows the results of the second part of the test, where in the problems the time was given and the body temperature was requested. Finally, the time calculation column corresponds to the third part of the evaluation, where the data is the body temperature and it is necessary to calculate the time at which the temperature is reached. Each section consisted of five questions, while the total duration of the evaluation was one hour.

Table 2. Averages of tests applied per indicator in each group.

| Group | Indicator of comprehension and application of concepts | | |
|------------------------|--|-------------------------|------------------|
| | System dynamics comprehension | Temperature calculation | Time calculation |
| A (traditional method) | 73 | 68 | 62 |
| B (proposed method) | 93 | 82 | 75 |

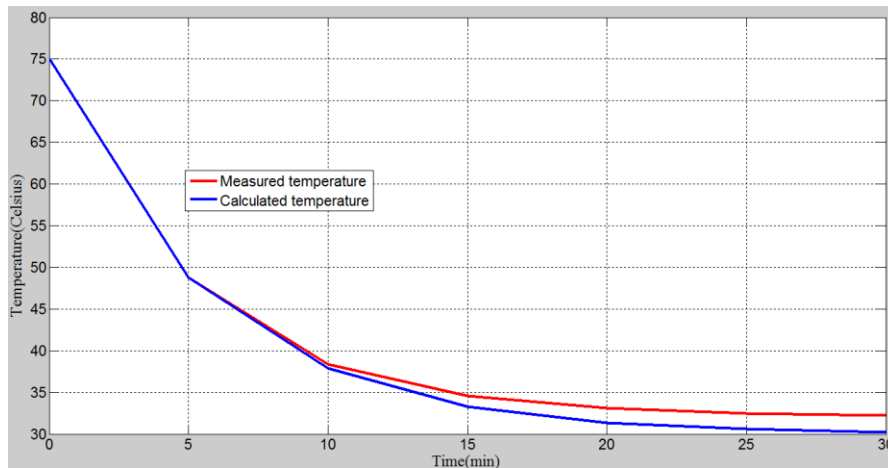


Fig. 2. Comparison between graphics of recorded and calculated temperature.

IV. CONCLUSION

The students were able to interpret that the results presented an important relationship between the graph of the temperature recorded and the theoretical solution curve of the differential equation, they also noticed that the discrepancies between them are in the most critical case of less than two degrees Centigrade, and that were generated due to the thermal radiation itself that affects the environment, altering the temperature thereof.

The ease of automatically collating the graphs allowed students to quickly observe that the rate of change in temperature of a body is directly proportional to the difference between that temperature and that of the environment. The results of the obtained evaluation indicate a greater academic performance in the students who verified the problem of cooling in the practice, especially in relation to the understanding of the dynamics of the system.

It is important to mention that these conclusions are based on the numerical values obtained from the evaluation instruments applied and shown in Table 2, but they not received rigorous statistical treatment to ensure that there were significant differences due to the application of the method. It is considering to be implemented in future researches.

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